New Approach to Solve Fuzzy Linear Programming Problems by the Ranking Function

A. Karpagam and Dr.P. Sumathi

Abstract--- *In this paper, a new method is proposed to find the fuzzy optimal solution of fully fuzzy linear programming problems with triangular fuzzy numbers. A computational method for solving fully fuzzy linear programming problems (FFLPP) is proposed, based upon the new Ranking function. The proposed method is very easy to understand and to apply for fully fuzzy linear programming problems occurring in real life situations as compared to the existing methods. To illustrate the proposed method numerical examples are solved.*

Keywords--- *Fully Fuzzy Linear Programming Problem, Fuzzy Number, Fuzzy Variable Linear Programming, Fuzzy Optimal Solution, Ranking Function, Triangular Fuzzy Number*

I. INTRODUCTION

THE concept of Fuzzy logic was first conceived by Loft Zadeh, a professor at the university of California at Berkley. But as a way of processing data by allowing partial set membership rather than crisp set membership or non-membership. Allahviranloo et al., (2008) [1] proposed a new method for solving fully fuzzy linear programming problems by the use of ranking function. Kumar et al., (2010) [2] proposed a new method for solving fully fuzzy linear programming problems with inequality constraints. Amit Kumar & Jagdeep Kaur, (2013) [3] proposed a new method for finding the fuzzy optimal solution of fully fuzzy linear programming problems with equality constraints. Bellman & Zadeh, (1970) [4] proposed the concept of decision making in fuzzy environment. Cadenas & Verdegay, (1997) [5] studied a linear programming problem in which all its elements are defined as fuzzy sets. (Chanas, 1983) [6] Proposed the possibility of the identification of a complete fuzzy decision in fuzzy linear Programming by use of the parametric programming technique. Fang et al., (1999) [7] presented a method for solving Linear programming problems with fuzzy coefficients in constraints. Jimenez et al., (2007) [8] proposed a method for solving linear programming problems where all the coefficients are fuzzy numbers and used a fuzzy ranking method to rank the fuzzy objective values and to deal with the inequality relation on constraints. Kaur & kumar (2012) [9]

proposed a new method for solving exact fuzzy optimal solution of fully fuzzy linear programming problems with unrestricted fuzzy variables

Kolman & Hill, (1984) [10] proposed a Fuzzy variable linear programming problems by use of a certain linear ranking function. Lotfi et al., (2009) [11] discussed fully fuzzy linear programming problems by representing all parameters and variables as triangular fuzzy numbers. Maleki et al., (2000) [12] solved the linear programming problems in which all decision parameters are fuzzy numbers by the comparison of fuzzy numbers. Nagoor Gani & Mohamed Assarudeen, (2013) [13] proposed a new operation on Triangular fuzzy number for solving fuzzy linear programming problem. Nasserri & Alizadeh, (2011) [14] proposed a method for solving fuzzy linear programming problems by solving the classical linear programming. Nehi et al., (2004) [15] defined the concept of optimality for linear programming problems with fuzzy parameters by transforming fuzzy linear programming problems into multiobjective linear programming problems. (Ramik, 2005) [16] proposed the fuzzy linear programming problems based on fuzzy relations. (Werners, 1987) [17] Introduced an interactive system which supports a decision maker in solving programming models with crisp or fuzzy constraints or fuzzy goals. Zhang et al., (2003) [18] proposed a method for solving fuzzy linear programming problems which involve fuzzy numbers in coefficients of objective functions. (Zimmerman, 1978) [19] proposed the though fuzzy logic has been applied to many fields, from at control theory to artificial intelligence, it still remains controversial among most statisticians, who prefer Bayesian logic, and some control engineers, who prefer traditional two-valued logic.

Today, Fuzzy logic concept used widely in many implementations like automobile engine and automatic gear control systems, air conditioners, automatic focus control, video enhancement in TV sets, washing machines, behavior-based mobile robots, sorting and handling data, Information Systems, traffic control systems and so on. In this paper, some preliminaries are presented in section II. Section III describes the existing methods. The section IV illustrates the new approach with one numerical example. Comparison results are presented in section V. Section VI concludes the paper.

II. PRELIMINARIES

Definition

The characteristic function μ_A of a crisp set $A \subset X$ assigns a value either 0 or 1 to each member in X . This function can be generalized to a function μ_A such that the value assigned to the element of the universal set X fall within a specified range

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i.e. $\mu_{\tilde{A}} : X \rightarrow [0,1]$. The assigned value indicate the membership function and the set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)); x \in X\}$ defined by $\mu_{\tilde{A}}(x)$ for $x \in X$ is called fuzzy set.

Definition

A fuzzy number $\tilde{A} = (m, \alpha, \beta)$ is said to be triangular fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 1 - \frac{m-x}{\alpha}, & m-\alpha \leq x < m, & \alpha > 0 \\ 1 - \frac{x-m}{\beta}, & m \leq x < m+\beta, & \beta > 0 \\ 0, & \text{otherwise} \end{cases}$$

Definition

A fuzzy number $\tilde{A} = (m, \alpha, \alpha)$ is said to be a symmetric triangular fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x+\alpha-m}{\alpha}, & m-\alpha \leq x < m, & \alpha > 0 \\ \frac{-x+m+\alpha}{\alpha}, & m \leq x < m+\alpha, & \alpha > 0 \\ 0, & \text{otherwise} \end{cases}$$

Definition

The concept of Fuzzy decision making was first proposed by Bellman & Zadeh, (1970) [4]. A linear programming problem is called fuzzy variable linear programming problem (FVLPP), if some of the parameters are crisp, and variables and right hand sides are fuzzy numbers. General form of FVLPP as follows:

$$\text{Max (min)} \tilde{z} = c_j \tilde{x}_j$$

$$\text{Subject to } A_{ij} \tilde{x}_j \leq \tilde{B}_i,$$

$$\tilde{x}_j \geq 0 \quad \text{Where } c_j \in R^n, A_{ij} \in (R)^{m \times n}, \tilde{B}_i \in (F(R))^m, \tilde{x}_j \in (F(R))^n$$

Definition

Kolman & Hill, (1984) [10] was introduced a FFLP problem. A linear programming problem is called fully fuzzy linear programming problem (FFLPP), if parameters and variables are all fuzzy numbers. General form of FFLPP as follows:

$$\text{Max (min)} \tilde{z} = \tilde{c}_j \tilde{x}_j$$

$$\text{Subject to } \tilde{A}_{ij} \tilde{x}_j \leq \tilde{B}_i,$$

$$\tilde{x}_j \geq 0$$

$$\text{Where } \tilde{c}_j \in (F(R))^n, \tilde{A}_{ij} \in (F(R))^{m \times n}, \tilde{B}_i \in (F(R))^m, \tilde{x}_j \in (F(R))^n$$

Definition

An effective approach for ordering the elements of $F(R)$ is also to define a ranking function $\Re: F(R) \rightarrow R$ which maps each fuzzy number into the real line, where a natural order exists. We define orders on $F(R)$ by:

$$\tilde{a} \geq \tilde{b} \quad \text{if and only if } R(\tilde{a}) \geq R(\tilde{b})$$

$$\tilde{a} > \tilde{b} \quad \text{if and only if } R(\tilde{a}) > R(\tilde{b})$$

$$\tilde{a} = \tilde{b} \quad \text{if and only if } R(\tilde{a}) = R(\tilde{b})$$

Definition

Let $\tilde{A} = (a_1, b_1, c_1)$ and $\tilde{B} = (a_2, b_2, c_2)$ be two triangular fuzzy numbers, then

$$(i) \quad \tilde{A} \leq \tilde{B} \text{ iff } a_1 \leq a_2, b_1 - a_1 \leq b_2 - a_2, c_1 - b_1 \leq c_2 - b_2$$

$$(ii) \quad \tilde{A} \geq \tilde{B} \text{ iff } a_1 \geq a_2, b_1 - a_1 \geq b_2 - a_2, c_1 - b_1 \geq c_2 - b_2$$

$$(iii) \quad \tilde{A} = \tilde{B} \text{ iff } a_1 = a_2, b_1 = b_2, c_1 = c_2$$

I. EARLIER METHOD

Amit kumar et.al., (2010) [2] has proposed to find the Fuzzy optimal solution of FFLP problems with inequality constraints by representing all the parameters are triangular fuzzy numbers.

$$\text{Maximize (Minimize)} (\tilde{C} \otimes \tilde{x})$$

$$\text{Subject to } \tilde{A} \otimes \tilde{x} \leq, =, \geq, \tilde{b}$$

Where $\tilde{x}$ is a non-negative triangular fuzzy number and

$$\tilde{C} = [\tilde{c}_j]_{1 \times n}, \tilde{A} = [\tilde{a}_{ij}]_{m \times n}, \tilde{x} = [\tilde{x}_j]_{n \times 1}, \tilde{b} = [\tilde{b}_i]_{m \times 1}$$

Example

$$\text{Max } (6, 4, 4) \tilde{x}_1 + (4, 4, 4) \tilde{x}_2$$

$$\text{Subject to } (4, 2, 2) \tilde{x}_1 + (2, 2, 2) \tilde{x}_2 \leq (10, 6, 6)$$

$$(6, 6, 6) \tilde{x}_1 + (8, 6, 6) \tilde{x}_2 \leq (8, 2, 2) \tilde{x}_1, \tilde{x}_2 \geq 0$$

Solution:

The standard form of given FFLPP

$$\text{Max } (6, 4, 4) \tilde{x}_1 + (4, 4, 4) \tilde{x}_2$$

$$\text{Subject to } (4, 2, 2) \tilde{x}_1 + (2, 2, 2) \tilde{x}_2 + (1, 1, 1) \tilde{s}_1 = (10, 6, 6)$$

$$(6, 6, 6) \tilde{x}_1 + (8, 6, 6) \tilde{x}_2 + (1, 1, 1) \tilde{s}_2 = (8, 2, 2)$$

$\tilde{x}_1, \tilde{x}_2, \tilde{s}_1, \tilde{s}_2$ are non-negative triangular fuzzy numbers

$$\text{Max } (6, 4, 4) (x_1, y_1, z_1) + (4, 4, 4) (x_2, y_2, z_2)$$

Subject to

$$(4, 2, 2) (x_1, y_1, z_1) + (2, 2, 2) (x_2, y_2, z_2) + (1, 1, 1) (s_1, t_1, u_1) = (10, 6, 6)$$

$$(6, 6, 6) (x_1, y_1, z_1) + (8, 6, 6) (x_2, y_2, z_2) + (1, 1, 1) (s_2, t_2, u_2) = (8, 2, 2)$$

$$\text{Max } \Re(6x_1 + 4x_2, 4y_1 + 4y_2, 4z_1 + 4z_2)$$

Subject to

$$(4x_1 + 2x_2 + s_1, 2y_1 + 2y_2 + t_1, 2z_1 + 2z_2 + u_1) = (10, 6, 6)$$

$$(6x_1 + 8x_2 + s_2, 6y_1 + 6y_2 + t_2, 6z_1 + 6z_2 + u_2) = (8, 2, 2)$$

$(x_1, y_1, z_1), (x_2, y_2, z_2), (s_1, t_1, u_1), (s_2, t_2, u_2)$ are non-negative triangular fuzzy numbers.

The above FFLPP is converted into the following Crisp linear programming:

$$\text{Max } \left(\frac{1}{4}\right) (12x_1 + 8x_2 + 4y_1 + 4y_2 + 4z_1 + 4z_2)$$

$$\text{Subject } 4x_1 + 2x_2 + s_1 = 10$$

$$6x_1 + 8x_2 + s_2 = 8$$

$$2y_1 + 2y_2 + t_1 = 6$$

$$6y_1 + 6y_2 + t_2 = 2$$

$$2z_1 + 2z_2 + u_1 = 6$$

$$6z_1 + 6z_2 + u_2 = 2$$

$$y_1 - x_1 \geq 0$$

$$y_2 - x_2 \geq 0$$

By simplex method, the Fuzzy optimal solutions are

$$\tilde{x}_1 = (1.33, 0.33, 0.33), \tilde{x}_2 = (0, 0, 0) \text{ and } \text{Max } \tilde{z} = 10.$$

III. PROPOSED METHOD

In this section, A new approach is proposed to find the Fuzzy optimal solution of fully fuzzy liner programming problems with symmetric triangular fuzzy numbers. The steps of the proposed method are as follows.

Algorithm

Step 1: Formulate the chosen problem into the following fully fuzzy linear programming problem:

$$\text{Max (min)} \tilde{z} = \tilde{c}_j \tilde{x}_j$$

$$\text{Subject to } \tilde{A}_{ij} \tilde{x}_j \leq \tilde{B}_i,$$

$$\tilde{x}_j \geq 0$$

Step 2: Using the Ranking function $\Re(\tilde{c}, \tilde{A}) = \frac{m+2\alpha-\beta}{2}$, the FFLPP transform into FVLPP.

Step 3: Solve the FVLPP by using simplex method / Big-M method. Let the solution be $\tilde{x}_j$. Hence the solution of FFLPP is $\tilde{x}_j^*$.

Step 4: Convert all the inequality constraints into equations by adding slack / surplus variable and the cost of this variable zero.

Step 5: Compute the value of $\tilde{Z} = C_B Y_j - C_j$ $j \neq B$ $j=1 \dots n$. If all $\tilde{Z} \geq 0 \forall j$ for maximization problem and $\tilde{Z} < 0 \forall j$ for minimization problem then the current solution is optimal, otherwise go to step 6.

Step 6: Determine the basic variable $\tilde{x}_k$, which will be replaced by the non-basic variable. Where $k = \text{argmin} \{ \Re(\tilde{B}) \}$, $i=1, 2, \dots, m$ in maximization problem and $k = \text{argmax} \{ \Re(\tilde{B}) \}$, $i=1, 2, \dots, m$ in minimization problem.

Step 7: Perform the pivot operation, return to step 5.

Example 4.1

$$\text{Max } (6, 4, 4) \tilde{x}_1 + (4, 4, 4) \tilde{x}_2$$

$$\text{Subject to } (4, 2, 2) \tilde{x}_1 + (2, 2, 2) \tilde{x}_2 \leq (10, 6, 6)$$

$$(6, 6, 6) \tilde{x}_1 + (8, 6, 6) \tilde{x}_2 \leq (8, 2, 2)$$

$$\tilde{x}_1, \tilde{x}_2 \geq 0$$

Solution:

Standard form

$$\text{Max } (6, 4, 4) \tilde{x}_1 + (4, 4, 4) \tilde{x}_2 + 0\tilde{x}_3 + 0\tilde{x}_4$$

$$\text{Subject to } (4, 2, 2) \tilde{x}_1 + (2, 2, 2) \tilde{x}_2 + (1, 1, 1)\tilde{x}_3 = (10, 6, 6)$$

$$(6, 6, 6) \tilde{x}_1 + (8, 6, 6) \tilde{x}_2 + (1, 1, 1) \tilde{x}_4 = (8, 2, 2)$$

$$\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4 \geq 0$$

Its equivalent FVLPP is

$$\text{Max } 5\tilde{x}_1 + 4\tilde{x}_2 + 0\tilde{x}_3 + 0\tilde{x}_4$$

$$\text{Subject to } 3\tilde{x}_1 + 2\tilde{x}_2 + \tilde{x}_3 = (10, 6, 6)$$

$$6\tilde{x}_1 + 5\tilde{x}_2 + \tilde{x}_4 = (8, 2, 2)$$

$$\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4 \geq 0$$

Table 4.1.1

Basis	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$	RHS	$\Re(\tilde{B}) = \frac{m+2\alpha-\beta}{2}$
$\tilde{x}_3$	3	2	1	0	(10, 6, 6)	8
$\tilde{x}_4$	6	5	0	1	(8, 2, 2)	5
$\tilde{z}$	-5	-4	0	0	(0, 0, 0)	

Here -5 is the most negative in $\tilde{z}$, so $\tilde{x}_1$ is an entering variable and $\min \{ \Re(10, 6, 6), \Re(8, 2, 2) \}$ is 5. So $\tilde{x}_4$ is a leaving variable and its pivotal element is 6.

Table: 4.1.2

Basis	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$	RHS
$\tilde{x}_3$	0	$-\frac{3}{6}$	1	$-\frac{3}{6}$	$(\frac{36}{6}, \frac{30}{6}, \frac{30}{6})$
$\tilde{x}_1$	1	$\frac{5}{6}$	0	$\frac{1}{6}$	$(\frac{8}{6}, \frac{2}{6}, \frac{2}{6})$
$\tilde{z}$	0	$\frac{1}{6}$	0	$\frac{5}{6}$	$(\frac{40}{6}, \frac{10}{6}, \frac{10}{6})$

Since $\tilde{Z} \geq 0$, the fuzzy optimal solution of the FVLP problem is $\tilde{x}_1 = (\frac{8}{6}, \frac{2}{6}, \frac{2}{6})$, $\tilde{x}_2 = (0,0,0)$. Therefore, the fuzzy optimal solutions of FFLPP is $\tilde{x}_1^* = (\frac{8}{6}, \frac{2}{6}, \frac{2}{6})$, $\tilde{x}_2^* = (0,0,0)$ and the fuzzy optimal value is $\text{Max}\tilde{Z} = ((\frac{40}{6}, \frac{10}{6}, \frac{10}{6})) = 10$

IV. RESULTS AND DISCUSSION

The results of the fully fuzzy linear programming problem, chosen in the example, Obtained by using the existing method and proposed method are shown in Table 5.1.

Table: 5.1

Sl. no	Number of constraints	Number of variables	Earlier method		Proposed method	
			Number of Iterations	Fuzzy optimal value $\tilde{z}$	Number of Iterations	Fuzzy optimal value $\tilde{z}$
1	2	2	9	10	2	10
2	2	2	10	53	2	53
3	2	2	10	62	2	62
4	3	3	18	154	2	154
5	3	3	14	216	2	216
6	4	4	17	75	3	75
7	4	4	16	112	3	112
8	5	5	20	214	4	214
9	5	5	19	198	6	198
10	6	6	26	327	8	327

It is obvious from the result: the existing method and proposed method the fuzzy optimal value are same.

V. CONCLUSION

In this paper a new method is proposed for solving the fuzzy optimal solution of FFLP problem transform into FVLP problems. The FFLP problem is converted into FVLP problem using new Ranking function. Ranking function is reasonable and effective for calculating the triangular weights of criteria. But the proposed method is less time consumption compared to previous method. Proposed method requires a less number of iterations. Therefore it is easier to solve fully fuzzy linear programming problem.

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