Dynamic Response Control of Base Isolated Plane Frame Structure Using Multiple Tuned Mass Dampers

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Abstract--- *Dynamic response of a base isolated multi-storey plane frame structure with Multiple Tuned Mass Dampers (MTMD) subjected to harmonic ground excitation and Mexico earthquake excitations are studied. A four-storey plane frame structure having three degrees of freedom (two translations along x, y -axes and one rotation about z- axis) at each node, with MTMD on its base is considered for study. Comparison of response is made in terms of displacements and forces in order to investigate the effectiveness of the MTMD. The response of base isolated structure with MTMD is found to be less in comparison to the corresponding response without MTMD, implying that the MTMD is effective in reducing forces and displacements of the base isolated structure.*

Keywords--- *Base Isolation, MTMD, Mexico Earthquake Excitation, Newmark's Method*

I. INTRODUCTION

THE main problem of the base isolation is that under certain excitations, structures suffer from exorbitant displacements at the base. An aseismic hybrid control system, which combines rubber bearings and passive Tuned Mass Damper (TMD) was proposed by Yang *et al.*, [1] to decrease the dynamic response of base isolated structures. The system is a combination of both approaches: shifting of the structural period according to base isolation concept, and the damping effect due to tuned mass damper technique. The main disadvantage of a single TMD is its sensitivity of the effectiveness to the error in the natural frequency of the structure and/or that in the damping ratio of the TMD. The effectiveness of a TMD is reduced significantly by the mistuning or the off-optimum damping in TMD. As a result, the use of more than one TMD with different dynamic

characteristics has been proposed in order to improve the effectiveness. Multiple Tuned Mass Dampers with distributed natural frequencies were proposed by Xu and Igusa [2], [3] and also studied by Yamaguchi and Harnpornchai [4], Jangid and Datta [5], Abe and Fujino [6], Abe and Igusa [7], Chunxiang [8] and Sadek *et al.*, [9]. The review shows that considerable work is carried out on the use of TMD and MTMD in reducing effectively the vibration of fixed base structure idealized as a single degree of freedom system. Tsai [10] and Palazzo and Petti [11], [12] studied the effect of TMD on the response of base isolated structure. In their study they had modeled the superstructure as a shear type building and TMD as a single degree of freedom system.

In the present paper, the effectiveness of MTMD in controlling the response of a base isolated multi-storey plane frame structures are investigated. A four-storey plane frame structure having three degrees of freedom (two translations along x, y -axes and one rotation about z- axis) at each node, with MTMD on its base is considered for study. Each TMD is modeled using a two-noded element having one translational degree of freedom at each node. MTMD with uniformly distributed frequencies are considered for this purpose. The force deformation behavior of the laminated rubber bearing (base isolation system) is modeled as linear. A step-by-step Newmark's method is adopted to solve the equation of dynamic equilibrium. The effectiveness of MTMD in suppressing the dynamic response of base isolated structure is determined by comparing the response of corresponding structure without MTMD.

II. ANALYSIS

The superstructure is divided into number of elements consisting of beams and columns. The beams and columns are modeled using two noded frame elements with three degrees of freedom at each node i.e., two translations along x and y-axes and one rotation about z-axis. For each element, the stiffness matrix, consistent mass matrix, and transformation matrix are obtained. The mass matrix and the stiffness matrix of each element from local direction are transformed to global direction as proposed by Paz [13]. The mass matrix and the stiffness matrix of each element are assembled by direct stiffness method to get the overall mass matrix, M, and overall stiffness matrix, K, for the entire superstructure. Knowing the overall mass matrix, M, and overall stiffness matrix, K, the frequencies for the superstructure is obtained using simultaneous iteration method. The superstructure time period

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is expressed as $T_s = 2\pi / \omega_s$, where ω_s is the fundamental natural frequency of the superstructure. The damping matrix for superstructure is obtained using Rayleigh's equation, $C = \alpha M + \beta K$, where α and β are the constants. These constants can be determined easily if the damping ratio for each mode is known. The overall dynamic equation of equilibrium for the entire superstructure can be expressed in matrix notations as

$$M\ddot{u} + C\dot{u} + Ku = f(t) \quad (1)$$

where M , C and K are the overall mass, damping, and stiffness matrices of size $3N \times 3N$, where N is the number of nodes. $\ddot{u}$,

$\dot{u}$ and u are the relative acceleration, velocity and displacement vectors with respect to ground and $f(t)$ is the nodal load vector. $u = u_1, v_1, \theta_{z1}, u_2, v_2, \theta_{z2}, \dots, u_N, v_N, \theta_{zN}$.

The nodal load vector due to earthquake is obtained using the equation

$$f(t) = -M I \ddot{u}_g(t) \quad (2)$$

Where M is the overall mass matrix, I is the influence vector of size $3N \times 1$, $\ddot{u}_g(t)$ is the ground acceleration. The resulted equation of dynamic equilibrium is solved using Newmark's method to obtain the displacements and acceleration at the nodes as explained in Chopra [14]. Owing to its unconditional stability, the constant average acceleration scheme (with $\beta = 1/4$ and $\gamma = 1/2$) is adopted.

2.1 Modeling of Laminated Rubber Bearing

The laminated rubber bearings generally exhibit linear force deformation behavior. In the present study, the force deformation behavior of the laminated rubber bearing is modeled as linear, with one translational degree of freedom (x direction) at each node. Damping for each bearing is calculated using the equation

$$C_b = 2\xi_b \sqrt{k_b M_t} \quad (3)$$

where ξ_b is the damping ratio of the bearing and M_t is the mass on each bearing. The lateral stiffness k_b and the isolation period (T_b) of the bearing is calculated using the equation

$$k_b = M_t \omega_b^2 \quad (4)$$

$$\text{and } T_b = 2\pi / \omega_b \quad (5)$$

where ω_b is the base isolation frequency. The stiffness and the damping of the bearings are added at corresponding global degrees of freedom to the overall stiffness matrix and overall damping matrix of superstructure.

2.2 Modeling of Multiple Tuned Mass Dampers

Each TMD is modeled using a two-noded element having one translational degree of freedom (x- direction) at each node. The natural frequencies of the MTMD are uniformly distributed around their average natural frequency. The natural frequency ω_j (i.e. $\omega_j = \sqrt{k_j / m_j}$) of the j^{th} TMD is expressed as

$$\omega_j = \omega_T \left[1 + \left(j - \frac{n+1}{2} \right) \frac{\beta}{n-1} \right] \quad (6)$$

and

$$\omega_T = \sum_{j=1}^n \omega_j / n \quad (7)$$

$$\beta = \frac{\omega_n - \omega_1}{\omega_T} \quad (8)$$

Where n is the total number of MTMD, ω_n and ω_1 are the natural frequency of n^{th} and first TMD respectively, ω_T is the average frequency of all the MTMD and β is the frequency range parameter of the MTMD.

As suggest by Xu and Igusa [2], the manufacturing of MTMD with uniform stiffness is simpler than those with varying stiffnesses. In this study, the distribution of natural frequencies of the MTMD is achieved by keeping the stiffness constant (i.e., with $k_1 = k_2 = k_n = k_T$), but allowing the mass of each TMD to vary.

The mass and the damping constant of the j^{th} TMD are expressed as

$$m_j = \frac{k_T}{\omega_j^2} \quad (9)$$

$$c_j = 2m_j \xi_T \omega_j \quad (10)$$

where ξ_T is the damping ratio which is kept constant for all the MTMD.

The ratio of total mass of MTMD to the total mass of the structure is defined as the mass ratio i.e.

$$\mu = \frac{\sum_{j=1}^n m_j}{m_s} = \frac{m_T}{m_s} \quad (11)$$

The constant stiffness required for each TMD can be evaluated as

$$k_T = \frac{\mu m_s}{\sum_{j=1}^n 1/\omega_j^2} \quad (12)$$

The ratio of average frequency of MTMD to the fundamental frequency of the base isolated structure is defined as tuning frequency ratio i.e.

$$f = \frac{\omega_T}{\omega_b} \quad (13)$$

The stiffness, damping and mass matrices of each TMD is expressed as

$$k_{TMD} = \begin{bmatrix} k_T & -k_T \\ -k_T & k_T \end{bmatrix} \quad (14)$$

$$c_{TMD} = \begin{bmatrix} c_j & -c_j \\ -c_j & c_j \end{bmatrix} \quad (15)$$

$$m_{TMD} = \begin{pmatrix} 0 & 0 \\ 0 & m_j \end{pmatrix} \quad (16)$$

The stiffness, damping and mass matrices of each TMD are added at corresponding global degrees of freedom to the overall stiffness matrix, overall damping matrix and overall mass matrix of base isolated structure.

2.3 Determination of Member Forces

The displacement obtained at each node is assigned for each member. The forces in each member are then obtained by multiplying element stiffness matrix with the nodal displacement vector.

III. RESULT AND DISCUSSION

3.1 Response of a Multi-Storey Base Isolated Plane Frame Structure with and without MTMD Subjected to Harmonic Ground Excitation

The response of a four-storey plane frame structure with MTMD on its base (Figure 1) subjected to harmonic ground excitation equal to $a_0 \sin(\omega t)$ (where a_0 is equal to 20% of acceleration due to gravity and ω is the input frequency) is studied. The displacements, base shear and the bending moment in the members due to this loading are computed.

Figure 1: A Base Isolated Four Storey Plane Frame Structure with MTMD on its Base

The material and geometric properties considered for the analysis is given in table 1. Figure 2(a), 2(b), 2(c) and 2(d) shows the variation of the maximum base displacement, top floor displacement, base shear and bending moment at bottom column respectively against the frequency ratio (ω/ω_b) for a base isolated structure with (4 TMD & 8 TMD) and without MTMD.

Table 1: The Details of Plane Frame

T_s (sec)	0.75
Mass on each beam (kN-sec ² /m ²)	5.775
B (m)	7
h(m)	3.5
Size of beam (m)	0.3 x 0.6
Size of column (m)	0.6 x 0.6
E (kN/m ²)	2.24×10^7

The damping ratio of superstructure is taken as 2% of critical for all modes, mass ratio is taken as 1%, tuning frequency ratio is taken as unity, damping ratio of the MTMD is taken as 1%, frequency range parameter of the MTMD is taken as 0.2, isolation damping is taken as 5% of critical and isolation period is taken as 2.0 sec. It is found that the peak value of the base displacement, top floor displacement, base shear and bending moment of a base isolated structure decreases due to MTMD. Hence the MTMD can be used effectively to control the response of a base isolated structure. Further, it can also be seen from the figures that, as the total number of TMD increase the effectiveness of MTMD in suppressing dynamic response of the structure increases. In case of 4 TMD, there exist significant secondary peaks, which are caused by resonances of the TMD's and that one of the secondary peaks gives the maximum response of the structure. Whereas in case of 8 TMD, the secondary peaks vanish and the maximum response of the structure occurs at the primary peak. It is also found that, in the frequency range, $0.8 > \omega/\omega_s > 1.25$, the response curves of structure with and without MTMD is almost same; this indicates that MTMD is effective only near the fundamental natural frequency of base isolated structure.

3.2 Response of a Multi-Storey Base Isolated Plane Frame Structure with and Without MTMD Subjected to Mexico Earthquake Excitation

The response of a four-storey plane frame structure subjected to Mexico earthquake excitation is studied. The base displacement, top floor displacement, base shear and bending moment at bottom column due to this loading are computed for a total period of 50.0 seconds. The time history of response is shown in figure 3(a) 3(b), 3(c) and 3(d). The damping ratio of superstructure is taken as 2% of critical for all modes, the total number of MTMD is taken as 8, mass ratio is taken as 1%, tuning frequency ratio is taken as unity, damping ratio of the MTMD is taken as 1%, frequency range parameter of the MTMD is taken as 0.2, isolation damping is taken as 5% of critical and isolation period is taken as 2.0 sec. It can be seen from figures that there is a considerable reduction in the base displacement, top floor displacement, base shear and bending moment due to MTMD. Hence the MTMD can be used effectively to control the seismic response of a base isolated structure.

Figure 2(A): The Base Displacement of a Base Isolated Four-Storey Plane Frame Structure

Figure 2(B): The Top Floor Displacement of a Base Isolated Four-Storey Plane Frame Structure

Figure 2(C): The Base Shear of a Base Isolated Four-Storey Plane Frame Structure

Figure 2(D): The Bending Moment of a Base Isolated Four-Storey Plane Frame Structure

Figure 3(A): The Base Displacement of a Base Isolated Four-Storey Plane Frame Structure

Figure 3(B): The Top Floor Displacement of a Base Isolated Four-Storey Plane Frame Structure

Figure 3(C): The Base Shear of a Base Isolated Four-Storey Plane Frame Structure

Figure 3(D): The Bending Moment of a Base Isolated Four-Storey Plane Frame

IV. CONCLUSION

The response of a multi-storey base isolated plane frame structure with MTMD on its base subjected to harmonic ground excitation and Mexico earthquake excitations are studied. The effectiveness of MTMD in suppressing the dynamic response of base isolated structure is determined by comparing the response of corresponding structure without MTMD. The response of base isolated structure with MTMD is found to be less in comparison to the corresponding response without MTMD, implying that the MTMD is effective in reducing forces and displacement of a base isolated structure.

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