Reliability Analysis of 2D Steel Frame

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Abstract--- *A two dimensional frame has been designed according to IS 800-2007 and are subjected to dead load, live load and earthquake load combination. In the present study reliability assessment of structural safety is studied. The uncertainties in geometry, loads and strength are accounted. The performance function for strength is studied using Hasofer Lind method. A MATLAB program has been developed for computing the reliability index for beams and columns. System reliability has been performed on the structure to find the reliability index of the system.*

Keywords--- *Reliability Analysis, Probability of Failure, Hasofer Lind Method, Reliability Index, System Reliability.*

I. Introduction

STRUCTURAL reliability analysis for systems plays an important role in the analysis and design of structures. The main purpose of structural reliability analysis is to evaluate the structural probability of failure. The probability of structural failure, takes into account the uncertainties associated with loads, geometry and resistance. Reliability assessment techniques help to develop safe designs and identify critical limit states. Structural reliability analysis results numerical measure of structural safety, given in terms of a failure probability or probability of safety.

A. Reliability Analysis

The failure of limit state performance is expressed as P_f, and the reliability as R follows the relation:

$$\text{Probability of safety (Reliability)}=1-P_f \quad (1)$$

According to IS 800:2007 the tension member has two modes of failure i.e:

i. Failure by Yielding
ii. Failure by Rupture

$$G(R, S)\leq 0 \quad (2)$$

$$P_f=P[G(R, S)\leq 0]=P[R-S<0]=P[Z\leq 0]$$

$$=\int f_{RS}(R,S)dRdS \quad (3)$$

$$P[Z\leq 0]= P[R-S<0] \quad (4)$$

$$\mu_z=\mu_R-\mu_S \text{ , } \sigma^2_z=\sigma^2_R-\sigma^2_S \quad (5)$$

where the G(r,s) is a limit function, f_{RS} is a joint density distribution of load and resistance (R is resistance and S is stress-load). Mean and standard deviation are marked as μ and σ.

II. Formulation of Steel Beam and Column

Strength formulation of steel Beam:

$$G(R, S)=R-S=F_{y.}\, Z_e-M \quad (6)$$

Where G(R, S) is limit function, F_y= Yield stress of steel in N/mm^2, Z_e= Sectional modulus in mm^3, M= Moment in N-mm

Force formulation of column in buckling:

$$G(R, S)=R-S=P_{cr}-P \quad (7)$$

Where

(Critical load)$P_{cr}= \pi^2EI/kL^2$

P= Actual load(external) in N,

E=Young's modulus in N/mm^2,

I= Moment of inertia in mm^4,

L= Length of column in mm,

k= Effective length factor

A. Hasofer Lind Method

The Hasofer-Lind method is applicable for normal random variables. It first defines the reduced variables as

$$X_i'=\frac{X_i-\mu_{Xi}}{\sigma_{Xi}}(i=1,2,..n) \quad (8)$$

Where, X_i' is a random variable with zero mean and unit standard deviation. Equation 8is used to transform the original limit state g(X) = 0 to the reduced limit state,

g(X') = 0. The X coordinate system is referred to as the original coordinate system. The X' coordinate system is referred to as the transformed or reduced coordinate system. The safety index, β is defined as the minimum distance from the origin of the axes in the reduced coordinate system to the limit state surface (failure surface). It can be expressed as

$$\beta=\sqrt{(X'^*)'(X'^*)} \quad (9)$$

The minimum distance point on the limit state surface is called the design point or checking point. It is denoted by vector x* in the original coordinate system and by vector X'* in the reduced coordinate system. These vectors represent the values of all the random variables, that is, X1, X2… Xn at the design point corresponding to the coordinate system being used.

This method can be explained with the help of Fig 1. Consider the linear limit state equation in two variables,

$$Z=R-S=0 \quad (10)$$

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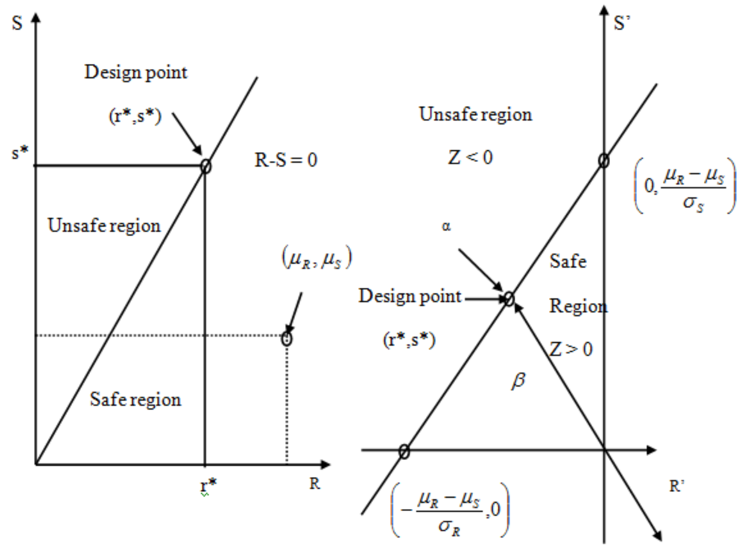
Note that R and S need not be normal variables. A set of reduced variables is introduced as

$$S' = \frac{S - \mu_S}{\sigma_S} \qquad (11)$$

$$R' = \frac{R - \mu_R}{\sigma_R} \qquad (12)$$



(a) Original Coordinates (b) Reduced Coordinates

Figure 1: Hasofer-Lind reliability index: Linear Performance Function

If we substitute these into Equation 10, the limit state equation in the reduced coordinate system becomes

$$g(X') = \sigma_R R' - \sigma_S S' + \mu_R - \mu_S = 0 \qquad (13)$$

The transformation of the limit state equation from the original to the reduced coordinate system is shown in Fig.1b. The safe and failure regions are also shown. From Fig.1b it is apparent that if the failure line (limit state line) is closer to the origin in the reduced coordinate system, the failure region is larger, and if it is farther away from the origin, the failure region is smaller. Thus, the position of the limit state surface relative to the origin in the reduced coordinate system is a measure of the reliability of the system. The coordinates of the intercepts of Equation 13 on the R' and S' axes can be shown to be $[-(\mu_R - \mu_S)/\sigma_R, 0]$ and $[0, (\mu_R - \mu_S)/\sigma_S]$, respectively.

Using simple trigonometry, we can calculate the distance of the limit state line (Equation 13) from the origin as

$$\beta = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \qquad (14)$$

An algorithm was formulated by Rackwitz (1976) to compute β and xi" as follows:

Step 1: Define the appropriate limit state equation.

Step 2: Assume initial values of the design point x_i^*, i =1, 2,…,n. Typically, the initial design point may be assumed to be at the mean values of the random variables. Obtain the reduced $x_i'^* = \frac{(x_i^* - \mu_{xi})}{\sigma_{xi}}$.

Step 3: Evaluate $\left(\frac{\partial g}{\partial X_i'}\right)^*$ and α_i at x_i".

Step 4: Obtain the new design point x_i" in terms of β, as in Equation 14.

Step 5: Substitute the new x_i" in the limit state equation g (x_i") = 0 and solve for β.

Step 6: Using the β value obtained in Step 5, evaluate β_{HL} x_i"= - $\alpha_i\beta$.

Step 7: Repeat steps 3 through 6 until βconverges.

B. System Reliability

There are basically two types of systems:

Series System

A system in which all components must be operating for the system to be successful is called a series system. Alternatively, the failure of any one component will cause the system to fail. The reliability of a series system is the probability that all the components in the system are successful. For n independent components, this is:

$$R = \prod_1^n (1 - p) \qquad (15)$$

Where p is probability of failure of individual components.

Parallel System

A system for which the success of any one component is equivalent to the success of the system is a parallel system. Alternatively, all the components must fail before the parallel system fails. The reliability of a parallel system is the probability that all of the components do not fail. Assuming independence, we have

$$R = 1 - \prod_{1}^{n}(p) \qquad (16)$$

Where p is probability of failure of individual components.

III. PROBLEM

Calculate the reliability index and sensitivity of variables of the components for a structure given in the figure below. Also find the system reliability of the structure.

Data:Dead load= 11.25kN,

Live load= 12kN, Super imposed dead load=4.5kN, H1=2.72kN (First floor)

H2=10.86kN (Second floor)

H3=24.46kN (Third floor)

H4=42.02kN (Fourth floor)

Fy=250N/sq.mm, E=200000N/sq.mm

Earthquake Zone V, Load combination: 1.2(DL+LL+ SIDL+EQX)

Special Moment Resisting Frame

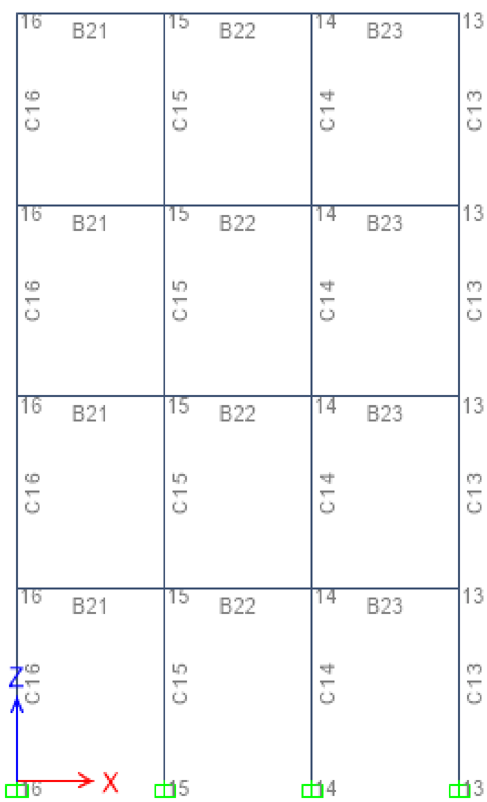


Figure 2: Model of the Structure

C. Elastic Analysis Results

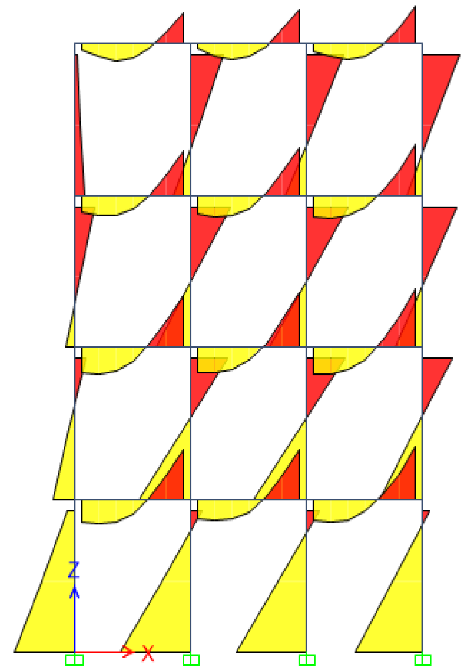


Figure 3: Bending Moment Diagram

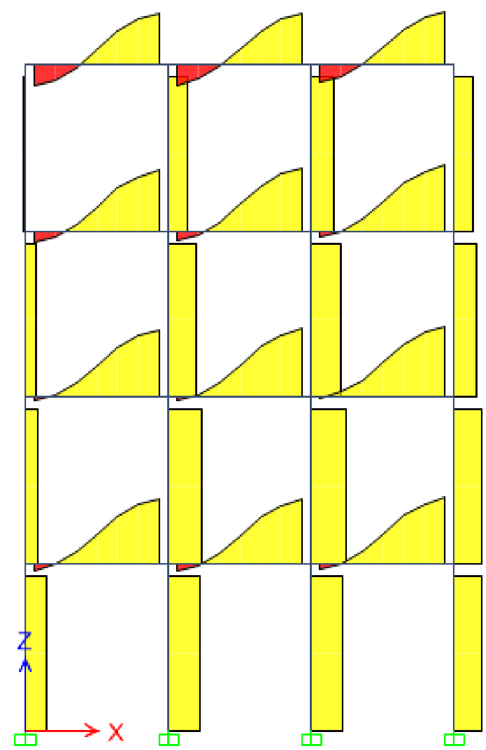


Figure 4: Shear Force Diagram

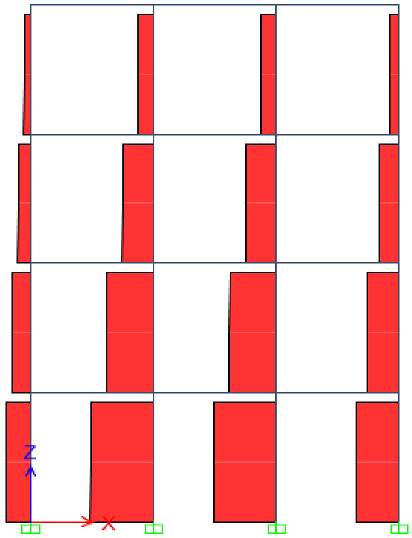


Figure 5: Axial Loads on Columns

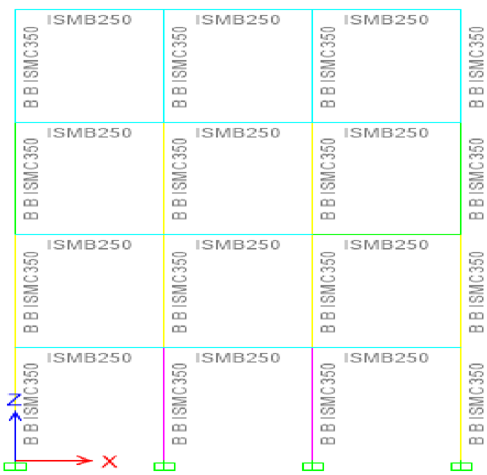


Figure 6: Steel Sections for Beam and Column

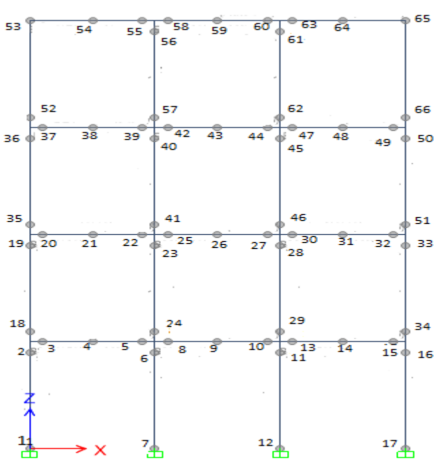


Figure 7: Expected Hinge Location

A. *Statistical Data of Random Variable*

Table 1: Statistical Data of Random Resistance and Geometrical Variables

Variable	*Mean*	*Standard Deviation*
Z (mm^3)	447290	0.05
F_y(N/mm^2)	272.405	0.05
L(column)(mm)	3813.674	0.05
E(N/mm^2)	2.179×10^5	0.05
A(column)(mm^2)	17151	0.05
r_{min} (column)	37.7	0.05

Table 2: Statistical Data of Load Variable

Floor No.	*Mean Bending Moment(kNm)*	*Mean Shear force(kN)*	*Mean Axial force(kN)*	*Standard deviation*
1	B21=28.44 B22=28.61 B23=30.14	B21=34.57 B22=34.07 B23=35.01	C13=263.74 C14=382.73 C15=386.23 C16=149.85	0.2
2	B21=30.40 B22=34.54 B23=33.76	B21=36.02 B22=36.31 B23=37.70	C13=196.60 C14=286.48 C15=288.53 C16=114.35	0.2
3	B21=25.35 B22=27.49 B23=29.77	B21=32.02 B22=33.27 B23=34.75	C13=122.69 C14=191.47 C15=192.42 C16=80.73	0.2
4	B21=18.31 B22=19.72 B23=21.15	B21=27.17 B22=27.45 B23=28.46	C13=60.98 C14=96.99 C15=97.71 C16=43.33	0.2

B. *Reliability Analysis results*

Reliability analysis of Steel Beam

Table 3: Tabulation of Reliability Index and Direction Cosine of Beam

Floor no.	*Beam no.*	β	α_1	α_2	α_3
1	B21 B22 B23	9.96 9.91 9.51	-0.52 -0.52 -0.51	-0.52 -0.52 -0.51	0.66 0.66 0.68
2	B21 B22 B23	9.44 9.15 8.62	-0.51 -0.51 -0.50	-0.51 -0.51 -0.50	0.68 0.69 0.7
3	B21 B22 B23	10.85 10.22 9.60	-0.54 -0.53 -0.51	-0.54 -0.53 -0.51	0.64 0.65 0.67
4	B21 B22 B23	13.26 12.73 12.21	-0.58 -0.57 -0.56	-0.58 -0.57 -0.56	0.56 0.58 0.6

Table 4: Sensitivity of Beam Variables

Floor no.	*Beam nos.*	Z_e (%)	F_y (%)	*M* (%)
1	B21 B22 B23	28 28 27	28 28 27	44 44 46
2	B21 B22 B23	26 26 25	26 26 25	48 48 50
3	B21 B22 B23	29 28 27	29 28 27	42 44 46
4	B21 B22 B23	33 32 31	33 32 31	34 36 38

C. Reliability analysis of Steel Column

Table 5: Tabulation of Reliability Index and Direction Cosine of Column

Floor no.	*Column nos.*	*β*	*α1*	*α2*	*α3*	*α4*	*α5*
1	C13	11.98	-0.24	-0.24	-0.76	0.34	0.41
	C14	10.36	-0.26	-0.26	-0.71	0.37	0.46
	C15	10.32	-0.26	-0.26	-0.71	0.37	0.46
	C16	14.12	-0.19	-0.19	-0.85	0.28	0.34
2	C13	13.14	-0.22	-0.22	-0.81	0.31	0.37
	C14	11.63	-0.24	-0.24	-0.75	0.35	0.42
	C15	11.60	-0.24	-0.24	-0.75	0.35	0.42
	C16	14.98	-0.17	-0.17	-0.88	0.25	0.30
3	C13	14.66	-0.18	-0.18	-0.87	0.26	0.32
	C14	13.24	-0.21	-0.21	-0.81	0.31	0.37
	C15	13.23	-0.21	-0.21	-0.81	0.31	0.37
	C16	15.91	-0.13	-0.13	-0.92	0.20	0.26
4	C13	16.50	-0.11	-0.11	-0.94	0.17	0.22
	C14	15.44	-0.15	-0.15	-0.90	0.23	0.28
	C15	15.42	-0.15	-0.15	-0.90	0.23	0.28
	C16	17.06	-0.09	-0.09	-0.96	0.13	0.18

Table 6: Sensitivity of Column Variables

Floor	*Column nos.*	*E(%)*	*A(%)*	*r_{min}(%)*	*L(%)*	*P(%)*
1	C13	5.76	5.76	57.76	11.56	16.81
	C14	6.76	6.76	50.41	13.69	21.16
	C15	6.76	6.76	50.41	13.69	21.16
	C16	3.88	3.88	72.25	7.84	11.56
2	C13	4.84	4.84	65.61	9.61	13.69
	C14	5.76	5.76	56.25	12.25	17.64
	C15	5.76	5.76	56.25	12.25	17.64
	C16	2.89	2.89	77.44	6.25	9.00
3	C13	3.20	3.20	75.69	6.76	10.24
	C14	4.41	4.41	65.61	9.61	13.69
	C15	4.41	4.41	65.61	9.61	13.69
	C16	1.69	1.69	84.64	4.00	6.76
4	C13	1.21	1.21	88.36	2.89	4.84
	C14	2.25	2.25	81.00	5.29	7.84
	C15	2.25	2.25	81.00	5.29	7.84
	C16	0.81	0.81	92.16	1.69	3.24

D. System Reliability Results

Twenty seven failure modes has been identified and reliability index was calculated for each mode. To get the worst probability of failure, the system was considered as a series system. Therefore Reliability of the system is:

$$R=\prod_{1}^{27}(1-0.00001)=0.99973$$

Probability of failure of the system:

$$Pf= 1-0.99973=0.00027$$

I. Load and Resistance Factor

Load and resistance factors are given by:

$$\text{Resistance factor}(\phi)= \exp(\alpha R^{*}\beta(cov)R)$$

Load factor(γ)=exp(αS*β(cov)S)

Where (cov)R= Covariance of Resistance,

(cov)S= Covariance of strength, β=Reliability index

Average load factor for beam= 3.87

Average resistance factor for beam=0.57

Average load factor for column=2.47

Average resistance factor for column=0.17

Table 7: Load and Resistance Factor for Beam

Floor no.	*Beam nos.*	*Load factor(γ)*	*Resistance factor(φ)*
1	B21	3.78	0.59
	B22	3.76	0.59
	B23	3.65	0.61
2	B21	3.63	0.61
	B22	3.54	0.62
	B23	3.38	0.64
3	B21	4.01	0.55
	B22	3.85	0.58
	B23	3.67	0.60
4	B21	4.52	0.46
	B22	4.43	0.48
	B23	4.33	0.50

Table 8: Load and Resistance Factor for Column

Floor	*Column nos.*	*Load factor(γ)*	*Resistance factor(φ)*
1	C13	2.67	0.20
	C14	2.59	0.24
	C15	2.59	0.24
	C16	2.61	0.15
2	C13	2.64	0.17
	C14	2.65	0.21
	C15	2.65	0.21
	C16	2.45	0.14
3	C13	2.55	0.14
	C14	2.66	0.17
	C15	2.66	0.17
	C16	2.28	0.13
4	C13	2.06	0.13
	C14	2.37	0.13
	C15	2.37	0.13
	C16	1.84	0.13

IV. CONCLUSION

1. The beam and column components designed according to IS 800:2007 are safe but uneconomical as target reliability index according to ISO is 3.5.
2. All the identified failure modes are safe (β>3.5).
3. Beams are more sensitive to loads and columns are sensitive to radius of gyration.
4. The bounds of system reliability ranges from $0<p_f<0.00027$.
5. The load factors are greater than unity and resistance factors are less than unity.
6. LRFD form:
   For beam 0.57R≥3.87S
   For column 0.17R≥2.47S

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