Analytical Study on Effective Width of Equivalent Frame Flat Plate Structural System Subjected to Seismic Loads

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Abstract--- *Flat plate structures are commonly used in moderate and low seismic zones as lateral force resisting systems. The study on flat plates showed that under seismic loading entire width and stiffness is not effective. There is difference of opinion as to what should be the stiffness of the beam slab for the analysis of equivalent frame for horizontal loads. In this paper attempt has been made to study the efficiency of the equivalent frame methods proposed by various researchers and carry out finite element analysis using computer program. The equivalent frame method proposed by various scientists has found suitable for approximate lateral load analysis; which results in conservative design. The study showed that under lateral load, the larger width of the slab participating in resisting the lateral loads and hence contributes larger stiffness to the structure. Hence finite element modelling is the good approach for the lateral load analysis. In equivalent frame method, Hwang and Moehle method found suitable for the analysis of lateral loads; comparing to British code, Grossman and Wallace method.*

Keywords--- *Flat Plate, Effective Width, FEM Modelling, Drifts, Displacements, Lateral Stiffness.*

I. INTRODUCTION

FLAT plate floor system is a two-way concrete slab supported directly on columns with reinforcement in two orthogonal directions.

Primarily used in commercial buildings, and hospitals, this system has the advantages of simple construction and formwork and a flat ceiling. The flat plates develop large bending moment & shear forces close to the columns. These stresses bring about the cracks in concrete & may provoke the failure of slab. Flat plate structures are commonly used in moderate and low seismic zones as lateral force resisting systems whereas they are coupled with shear walls or moment resisting frames in high seismic zones. Difficulty faced in usage of this flat-plate system is in its behavior analysis, especially relate to the effect of lateral loading when the structure is designed to resist strong wind or earthquake. Static analysis with classical mechanics is not adequate because of the complex stress distribution that occurred in these plates. Advance computer technology development lead to fast progress in solving the problem numerically. The numerical method most common used in structural analysis is the Finite Element Method. The structural model widely used in structural analysis is the frame element model. This model is often preferred because simplicity and ease application. In the flat-plate structure an analysis by using frame elements is generally carried out by the effective beam-width model. In this method, the plate is modeled as an equivalent beam with a certain effective width.

Structural engineers commonly use the equivalent frame method with equivalent beams such as the one proposed by Jacob S. Grossman in practical engineering for the analysis of flat plate structures. The equations proposed by various researchers are containing modification factors and stiffness reductions factors to account the cracking of flat plate slabs. The efficiency of the various equivalent frame method proposed by Grossman, British code, Hwang, Kang and Moehle methods has been studied. The project involves the study of five models with different effective widths and stiffness reduction factors proposed by above mentioned researchers. The four models used for the study of equivalent frame method and one model is modeled using finite element method. The computer modelling has been done using a software ETABS. The equivalent frames are modeled as frame section element in the computer and finite element modelling is done by plate element. The regular, symmetric type of building structure has been considered for the study. The loads are calculated as per Indian standard codes. The effective width of equivalent slab-beam is computed and stiffness reduction factor considered in the study. The drifts, displacement, lateral stiffness and dynamic characteristics of structure have been studied for equivalent frame method and compared with the results of the finite element method.

II. LITERATURE REVIEW

The research study on flat plate structural system has showed that a small portion of slab width is effective in resisting the seismic loads. Several researchers have made study on the width of flat plate frames and proposed various equations based on their study and observations. The investigation included the study of drift, displacement, lateral stiffness and also dynamic characteristics of the building. The accounting for cracking also considered in the investigation. The provision of shear studs and bands improved the performance of the flat plate system under seismic loads by controlling the drifts, displacements and enhancing the ductility characteristics of the building. The effect of gravity load also predominant in flat plate structural system; the test

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indicated that the models subjected to gravity load can withstand higher lateral drift and maximum unbalanced moments. The study also showed that high reinforcement ratio increases the lateral stiffness and also effective width factor. The research study [11] under free vibration test and shake table test was indicated that the flat plate damage significantly under lateral loads. The equivalent width factor and cracking factor determined from the test were found good agreement between analytical and experimental models. In Grossman's study [6] the slab-column frames were modeled based on equivalent slab-beam concept. An equivalent slab-beam is flexural member having a rectangular section with its width and depth dimensions equal to effective slab width and the total slab thickness. The effective slab width was determined by considering slab-column geometry.

Grossman's model generally predicts smaller effective slab widths than the other models. Hwang and Moehle [3] presented the effective slab-beam width by using finite element techniques. The solutions apply to interior edge and corner connections, for column with square cross sections, and for combinations of ratio of column size to effective span. According to Hwang and Moehle, the influence of section crakes is accommodated by means of reduction factor to the element stiffness.

III. Problem Formulation

In this project work, attempt has been made to study the efficiency of the equivalent frame methods proposed by Grossman, British code, Kang and Wallace and Hwang and Moehle; and simulate the effective width of the slab by analyzing different models with the equations provided by the researchers and carry out finite element analysis using computer program to predict the equivalent effective width. The objective of the study includes analyzing the drift, displacements, lateral stiffness and effect of stiffness reduction factor. The study is limited to understand the seismic behavior of the structure under elastic analysis.

IV. Modelling and Analysis

The modelling and analysis of flat plates is carried out by a computer program ETABS. The geometry of the structure is symmetric and regular in nature. The equivalent frames are modeled as equivalent beam-column element in the computer model. In the finite element method, the entire width of the slab is modeled das plate element. The analysis of models is carried out for both static load cases as well as dynamic load cases. For lateral load cases, the equivalent static method and Response Spectrum analysis is used. The seismic zone factor III is considered for the analysis. The effective width of equivalent slab-beam is calculated by Grossman, British code, Kang and Wallace, and Hwang and Moehle method; since these equations are practically viable.

A. Geometry Details of the Structure

The geometry of the building consist 5m bay width in each direction and height of 3m. The size the column is 600mmx60mm. The thickness of the flat plate is kept 200mm for all floors. The total height of building is 15m. The table I show the geometry details of the structure.

Table 1: Geometry Details

Bay width	5m
Number of bays in -X- direction	5
Number of bays in -Y- direction	5
Floor height	3m
No of floors	4
Building height	15m
Thickness of flat plate slab	200mm
Size of the column	600x600mm

B. Grossman's Method

Grossman concluded that the flat plate system has a good resistance capacity for the lateral loads as well as gravity loads provided a proper detailing in the joint between the column and the slab and a new formula for the effective width was proposed by Grossman's shown in Eq. (1) by

$$b_e = \left[0.30 l_1 + C_1\left(\frac{l_2}{l_1}\right) + \frac{(C_2 - C_1)}{2}\right]\left(\frac{d}{0.9h}\right)(K_{FP}) \quad (1)$$

Where,

l1 = length of span of supports in direction parallel to lateral load

l2 = length of span of supports in direction transverse to lateral load

C1 = size of support in direction parallel to lateral load

C2 = size of support in direction transverse to lateral load

d = effective depth of slab h = slab thickness

KFP = factor adjusting 'be,' at edge exterior and corner supports (1.0 for interior supports, 0.8 for exterior and edge supports, 0.6 for corner supports)

Effective width Computation

Column size = 0.60mx0.60m, C1=C2 = 0.60 l1 =l2 =5m, (d/0.9h) = (170/0.9x200) = 0.94

Effective width for interior Panel

$$b_e = [0.30 \times 5 + 0.60 \times 1 + 0](0.94)(1) = 1.97m$$

Effective width for exterior edge

$$b_e = [0.30 \times 5 + 0.60 \times 1 + 0](0.94)(0.80) = 1.58m$$

Effective width for corner edge

$$b_e = [0.30 \times 5 + 0.60 \times 1 + 0](0.94)(0.60) = 1.18m$$

C. British Code Method

In this model the effective width is calculated by one-half of the bay width. This recommendation is provided by BS 8110. The stiffness reduction factor is not applied in this model. Therefore effective width is given by

Where,

l1, l2= Bay width of the frame

Effective width for interior panel:

$$b_e = \left(\frac{5}{2}+\frac{5}{2}\right)\times\frac{1}{2} = 2.50m$$

Effective width for exterior panel:

$$b_e = \left(\frac{5}{2}+\frac{0}{2}\right)\times\frac{1}{2} = 1.25m$$

D. Kang and Wallace Method

Thomas H.K. Kang and John W Wallace studied the behavior of reinforced concrete and post tensioned flat frames under shake table test. The α (equivalent width factor) value 0.75 and β (cracking factor) 0.33 values determined from the experimental study were found good agreement between analytical and experimental models. They proposed a new formula for the effective width shown in Eq. (3) for the equivalent frame method.

$$be = \alpha\, \beta\, l (3)$$

Where,

= modification factor for effective width,

= 0.75 for RCC slabs

= modification factor for cracking,

= 0.33 for RCC slabs

l = length of span of supports in direction parallel to lateral load

Effective width for interior panel:

$$b_e = 0.75\times 0.33\times 5 = 1.23m$$

Effective width for exterior edge:

$$b_e = 0.75\times 0.33\times 2.5 = 0.61m$$

E. Hwang and Moehle Method

Hwang and Moehle present the effective beam width by using finite element techniques. The solutions apply to interior, edge, and corner connections, for column with square cross sections,. The results are divided into two groups: one for the interior frame, and another for the exterior frame.

The variations of effective beam width "*be*" for an interior frame, which includes interior connections, can be represented as

$$b_e = \left(2C_1 + \frac{l_1}{3}\right) \quad (4)$$

The effective beam width for an exterior frame, which includes corner connections and edge connections, can be represented as

$$b_e = \left(C_1 + \frac{l_1}{6}\right) \quad (5)$$

Where,

C1 = Column width in the direction of lateral load

l1= length of span of supports in direction parallel to lateral load

According to Hwang and Moehle, the influence of section cracks is accommodated by means of a reduction factor to the element stiffness, taken as 1/3.

Effective width for interior panel:

$$b_e = \left(2\times 0.60 + \frac{5}{3}\right) = 2.86m$$

Effective width for interior panel:

$$b_e = \left(0.60 + \frac{5}{6}\right) = 1.43m$$

F. Tabulation of Computed Effective Width

The effective width calculated by using above equations has been tabulated in the table II.

Table 2: Tabulation of Effective Width

Sl No	*Method*	*Effective width (m)*		*Stiffness Reduction Factor*		
		Inter -ior	*Exter-ior*	*Mod el 1*	*Mod el 2*	*Mod el 3*
1	British Standard method	2.50	1.25	1	0.25	0.33
2	Kang and Wallace method	1.20	0.60	1	0.25	0.33
3	Hwang and Moehle method	2.80	1.40	1	0.25	0.33
4	Grossman's method	1.90	1.40	1	0.25	0.33
5	Finite element model	-	-	1	0.25	0.33

G. Finite Element Modelling of Flat Plate

It is necessary to use a refined finite element model to understand the true behavior and accurate stress distribution in the slab. In this model analysis of flat plate structure is done using the finite element method with the help of computer program ETABS. The flat plate structure is modeled as plate element in the computer program to understand behavior of the structure. The flat plate structure is divided into suitable finite element plates. This model is considered as bench mark model for comparing results obtained from the other equivalent frame analysis proposed by the various researchers. Since the flat plate element is modeled as plate element, the results obtained indicate true behavior of the structure under seismic loading. By comparing the displacement, drift and stiffness of the finite element model with the equivalent frame methods: It is possible to verify the efficiency of the equivalent frame methods and to establish a conclusion that, which method of equivalent frames method is suitable under seismic loading.

V. Results and Discussions

Results obtained from the analysis have been studied, discussed and presented in the form of tables and graphical modes. Displacement, drift, and story stiffness with respect to story of the building have been drawn to understand the behavior of the building. The comparison between different methods has been studied. The comparison of the finite element model is done with the other equivalent frame methods.

A. Base Shear Characteristics

The base shear obtained by the four equivalent frame methods has been compared with the finite element analysis model method. The base shear from response spectrum analysis reduced as the stiffness reduction factor introduced in the model. Increase in the stiffness reduction factor decreases the dynamic base shear and results in higher value of scale factor (Vst/Vdy).The maximum static and dynamic base shear is found for FEM method. The base shear obtained by Hwang and Moehle method varies 4% with respect to the static base shear of the FEM and a variation of 26% in dynamic base shear. The other method results indicate more percentage variation of base shear in static as well as dynamic base shear. The results indicate that Hwang and Moehle method indicates least variation of base shear with respect to the base shear of the FEM method. The maximum variation in base shear has been found for Kang and Wallace method. Table III and IV show the numerical results of base shear.

Table 3: Percentage Variation of Static Base Shear

Sl. No	*Methods*	*Static Base Shear, kN*	*% variation of base shear w.r.t. FEM*
1	British method	2661	7
2	Grossman's method	2544	11
3	Kang & Wallace method	2285	20
4	Hwang and Moehle method	2755	4
5	FEM	2872	-

Table 4: Percentage variation of dynamic base shear

Sl. No.	*Methods*	*Dynamic Base shear, kN*					
		Stiffness reduction factors					
		% variation of base shear w.r.t. FEM					
		1	%	0.25	%	0.33	%
1	British method	776	30	706	29	686	29
2	Grossman's method	716	35	658	34	639	33
3	Kang & Wallace method	569	49	528	47	513	47
4	Hwang and Moehle method	822	26	745	25	723	25
5	FEM	1107	-	997	-	960	-

B. Displacement Characteristics

The displacements have been obtained for four equivalent frame method models and for finite element method. The displacement is found maximum for Kang and Wallace method for without stiffness reduction factor and also with stiffness reduction factor. The variation of displacement found directly proportional effective width. The least value of displacement is obtained for FEM method and the next least displacement is found for Hwang and Moehle method; for models without stiffness reduction and with stiffness reduction. The displacement of the storey increases as the stiffness of equivalent beam-slab is reduced. The maximum displacement is found for Kang and Wallace method. The results show that there is not much variation in the magnitudes of displacement in British, Grossmans and Moehle method. Hwang and Moehle method provide more stiffness to the structure; since the equation provides maximum effective width of equivalent beam-slab; and hence displacement is less compared to other methods. The figure 5.1 shows the displacement envelope plot for all the methods with and without stiffness reduction factors. From the figure 1, it is observed that the displacements in Hwang and Moehle method without stiffness reduction factor have been found nearest to FEM method. Hence Hwang and Moehle equivalent frame method can be used as a suitable method for seismic analysis. Table 5 shows the displacements of the storey.

Table 5: Displacements of Storey for Various Models

Displacement(mm) for w/o stiffness reduction factor					
Storey	British	Gross	Kang	Moehle	FEM
4	26.70	28.60	37.10	25.40	15.50
3	20.00	21.20	26.60	19.20	12.20
2	11.80	12.40	15.10	11.40	7.60
1	4.00	4.10	4.80	3.90	2.80
Displacement(mm) for stiffness reduction factor 0.25					
4	32.40	34.70	44.20	30.90	18.30
3	23.80	25.30	31.30	22.90	14.30
2	13.80	14.50	17.40	13.40	8.80
1	4.50	4.70	5.40	4.40	3.10
Displacement(mm) for stiffness reduction factor 0.33					
4	34.90	37.40	47.30	33.30	19.60
3	25.50	27.00	33.20	24.50	15.10
2	14.70	15.40	18.30	14.20	9.20
1	4.80	4.90	5.70	4.70	3.30

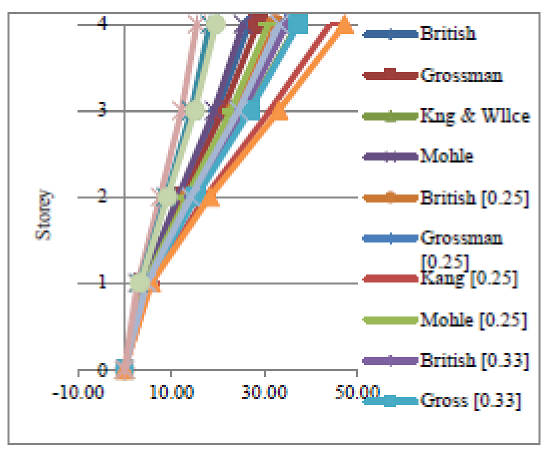

Figure 1: Envelope plot for Displacement- Storey

C. Driftt Characteristics

The drifts have been obtained for four equivalent frame method models and for finite element method. The drift is found maximum for Kang and Wallace method for without stiffness reduction factor and also with stiffness reduction factor; since effective width is less compared to other methods. Similar to the displacements, the variation of drift found directly proportional effective width. The least value of drift is obtained for FEM method and the next least drift is found for Hwang and Moehle method; for models without stiffness reduction and with stiffness reduction. The drift of the storey increases as the stiffness of equivalent beam-slab is reduced. The maximum drift is found for Kang and Wallace method. The results show that there is not much variation in the magnitudes of drift in British, Grossmans and Moehle method. Hwang and Moehle method provide more stiffness to the structure; since the equation provides maximum effective width of equivalent beam-slab; and hence drift is less compared to other methods. The figure 2 shows the displacement envelope plot for all the methods with and without stiffness reduction factors. From the figure 2, it is observed that the drifts in Hwang and Moehle method without stiffness reduction factor is been found nearest to FEM method. Hence Hwang and Moehle equivalent frame method can be used as a suitable method for seismic analysis.

Table 6: Displacements of Storey for Various Models

Drifts for w/o stiffness reduction factor					
Storey	British	Gross	Kang	Moehle	FEM
4	0.00227	0.00251	0.00364	0.00211	0.00110
3	0.00273	0.00295	0.00386	0.00259	0.00153
2	0.00261	0.00276	0.00342	0.00251	0.00162
1	0.00133	0.00137	0.00160	0.00129	0.00092
Drifts for stiffness reduction factor 0.25					
4	0.00291	0.00320	0.00451	0.00271	0.00136
3	0.00334	0.00360	0.00463	0.00318	0.00183
2	0.00309	0.00327	0.00398	0.00298	0.00188
1	0.00151	0.00156	0.00181	0.00148	0.00104
Drifts for stiffness reduction factor 0.33					
4	0.00320	0.00351	0.00489	0.00299	0.00148
3	0.00362	0.00388	0.00495	0.00344	0.00196
2	0.00330	0.00348	0.00422	0.00318	0.00200
1	0.00159	0.00164	0.00190	0.00156	0.00108

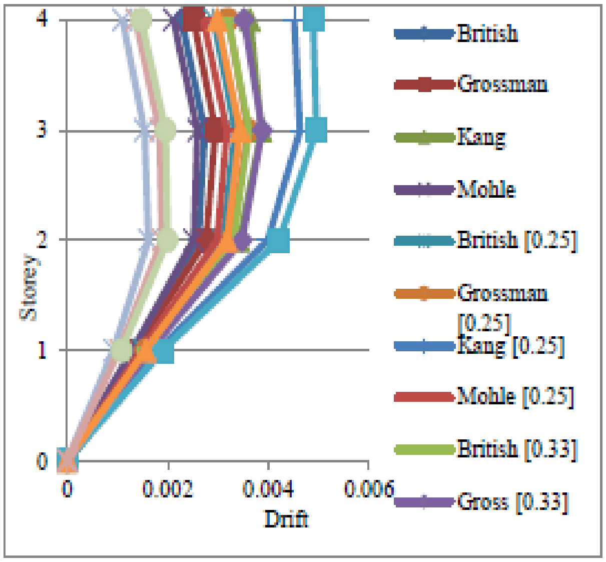

Figure 2: Envelope Plot for Drift- Storey

D. Lateral Stiffness Characteristics

The lateral stiffness has been obtained for four equivalent frame method models and for finite element method. The lateral stiffness is found maximum for FEM method for without stiffness reduction factor and also with stiffness reduction factor. The variation of lateral stiffness found directly proportional effective width. The least value of lateral stiffness is obtained for Kang and Wallace; and the next least lateral stiffness is found for Grossman method; for models without stiffness reduction and with stiffness reduction. The results show that there is not much variation in the magnitudes of lateral stiffness in British, Grossmans and Moehle method. Hwang and Moehle method provide more stiffness to the structure; since the equation provides maximum effective width of equivalent beam-slab; compared to other methods. The figure 3 shows the lateral stiffness envelope plot for all the methods with and without stiffness reduction factors

The lateral stiffness is found maximum for FEM model, since the entire slab width modeled ad plate element, the maximum portion of the width is effective in resisting lateral loads. From the results we can understand that modelling of flat plate by equivalent frame method by using Hwang and Moehle equivalent frame method is found suitable under seismic loading. The table 5 shows that lateral stiffness of the structure for various stiffness reduction factors. From the figure 3, it is observed that the stiffness in Hwang and Moehle method without stiffness reduction factor is been found nearest to FEM method. Hence Hwang and Moehle equivalent frame method can be used as suitable method for seismic analysis.

Table 7: Lateral stiffness for various models

Lateral stiffness for w/o stiffness reduction factor					
Storey	British	Gross	Kang	Moehle	FEM
4	205339	177261	111120	228859	454138
3	269749	238939	163560	294493	517474
2	328604	296950	217209	353588	572744
1	678266	625206	488108	718252	1038474
Lateral stiffness for stiffness reduction factor 0.25					
4	160263	138872	89437	178133	367506
3	220252	195651	136452	239942	433331
2	277492	251518	186818	297938	491056
1	593216	547617	431535	627452	924057
Lateral stiffness for stiffness reduction factor 0.33					
4	145742	126573	82528	161739	338587
3	203747	181277	127520	221712	404845
2	260116	236088	176542	278985	463199
1	563319	520435	411879	595483	883624

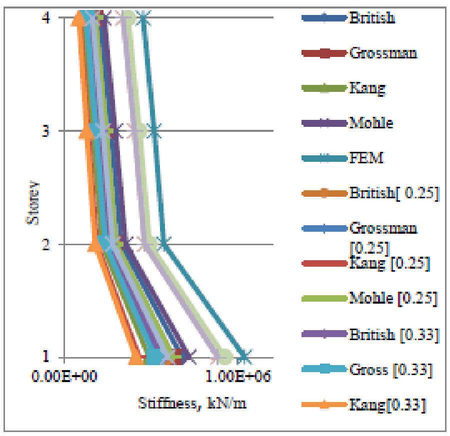

Figure 3: Envelope plot for Lateralstiffness- Storey

E. E. Natural Time Period characteristics

The time period of the structure is obtained for all models. The maximum time period has been found for Kang and Wallace method; and the least time period has been found for finite element model for without stiffness reduction factor as well with stiffness reduction factor. There is no much variation in the time period obtained by Grossman, British code and Hwang method. The reduction in the stiffness of the equivalent slab-beam makes the structure more flexible, thus increasing the time period of the structure. The finite element modelling considers the larger width of the slab participation to resist the lateral loads, thus increasing stiffness and reduction the time period value; thus the structure behave more rigid than the other equivalent frame methods. From the figure 4, we can note that the increase in the stiffness reduction factor increases the time period of the building, thus making structure more flexible. The Hwang and Moehle method gives the nearest value of time period to the finite element method.

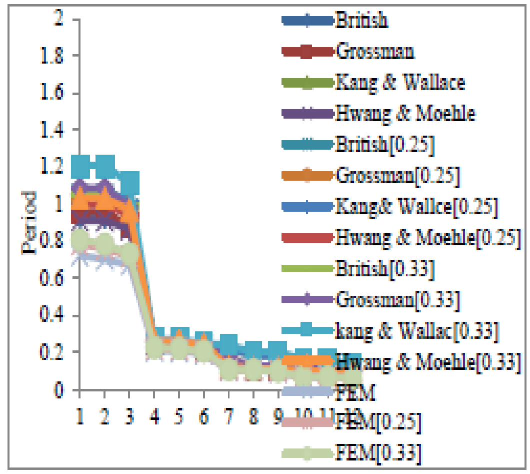

Figure 4: Envelope Plot for Mode-Time Period

VI. CONCLUSION

Based on the study and above observations, the following general conclusions have been drawn.

The equivalent effective width proposed by Grossman, Kang and Wallace, Hwang and Moehle, is on the conservative side. Hence these methods found suitable for approximate lateral load analysis. There is no much considerable variation in the displacements, drifts and lateral stiffness obtained by Grossman, British code, and Hwang and Moehle method; hence any of the method can be used for lateral load analysis. Hwang and Moehle method or BS 8110 method of computing equivalent slab-beam width for stiffness calculation can be considered as very acceptable method for analysis of horizontal loads. The lateral stiffness of the structure increases as the effective width of equivalent slab-beam section increases. Reduction in the stiffness of the slab-beam section decreases the stiffness of the structure and hence increases the displacement and drifts of the structure. The displacement and drift increase as the effective width decreases. The accounting for the cracking of the slab is not required in equivalent frame analysis; in case of moderate height buildings. Displacement increases, as the height of the building increases. The finite element modelling of the structure provides the higher stiffness to the structure; hence displacement and drift are considerably less. The study showed that under lateral load, the larger width of the slab participates in resisting the loads and hence contributes larger stiffness to the structure.

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