An Analytical Investigation on Behavior of Cross (+) Shaped RC Columns Using P_U-M_U Interaction Diagrams

Shanmukha Shetty, R.M. Subrahmanya, B.G. Naresh Kumar and H.S. Narashimhan

Abstract--- *The design aid for reinforced concrete to IS 456-1978, provides P-M interaction diagrams as a design aid to simplify the design of rectangular and circular columns. As these two sections are symmetric, it becomes comparatively easy to provide the respective interaction diagrams, whereas the same is difficult for nonconventional sections like flanged and tubular cross sections. In this work an effort has been put forth in this area to develop interaction diagram for cross(+) shaped columns by varying some of its parameters such as spacing of reinforcement, grade of concrete and steel, the results showed that with the increase in grade of material there was a considerable increase in load and moment carrying capacity of the column, also by varying the spacing higher strength could be achieved in comparison with equally spaced section for a given percentage of steel.*

Keywords--- *P_U –M_U Interaction Diagram, Cross (+) Shaped RC Columns, Behavioral Analysis*

I. INTRODUCTION

THE design of reinforced concrete columns is a tedious, time consuming, and iterative procedure, to simplify the design procedure SP-16 has provided non dimensional interaction diagrams for various d'/D ratio and percentage of steel. And SP-16 has also specified the procedure to develop interaction diagrams for any type of column with various specifications.

An interaction diagram could be drawn for a column with given specifications such as grade, percentage of steel, reinforcement detailing pattern, dimension and shape of column by evaluating strain at various locations in the column considered, there by evaluating stress which intern provides load and moment values, such numerous values of ultimate load and moment resisting capacity could be evaluated at various locations once the set of ultimate load and moment resistance values are obtained it could be graphically represented as an interaction diagram which serves as a failure envelop implying that any combination of load and moment that falls within the envelop is safe with the load-moment combination that falls on the envelop as the maximum load-moment resisting capacity of the section. Therefore interaction diagrams are of paramount importance in design of columns.

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II. SCOPE AND SIGNIFICANCE OF THE RESEARCH WORK

RC columns are usually subjected to axial loads in combination with bending in uni-axial or bi-axial directions for all practical cases, therefore the column rather than behaving as a pure compression member, behaves as abeam-column.SP-16:" Design aid for reinforced concrete to IS 456-1978" has provided various interaction diagrams for conventional column sections only and not for specially shaped sections, and there is a lack of information on the behavioral description of such specially shaped columns in terms of interaction diagrams in the code, therefore the present study is an attempt to develop interaction diagrams for one of the specially shaped RC columns i.e. cross (+) shaped columns, and study the influence of varying grade of material and reinforcement spacing on strength characteristics of cross (+) shaped columns, in course of the work a spread sheet has been developed using Microsoft excel which could supply interaction diagrams for cross (+) shaped, L-shaped, T-shaped, and rectangular columns with a provision to vary all specifications such as grade of concrete and steel, reinforcement spacing etc…these interaction diagrams would considerably makes the design process simpler and quicker.

III. INTERACTION DIAGRAMS

The interaction curve for a column is a graphical representation of design strength of columns subjected to uniaxial eccentricity for the given specifications. Axial load versus moment interaction diagram can be used for the design of eccentrically loaded columns, an interaction curve can be plotted by evaluating various axial load and moment combinations obtained for various positions of neutral axis.

Any point on the interaction curve has its co-ordinates as the ultimate load P_{UR} and ultimate moment M_{UR} as the maximum combination of load and moment that could be resisted by the column corresponding to a particular value of eccentricity, the line joining the origin 'O' to any point on the curve is called as the loading line, and reciprocal of slope of this line is called as eccentricity 'e' as shown in figure

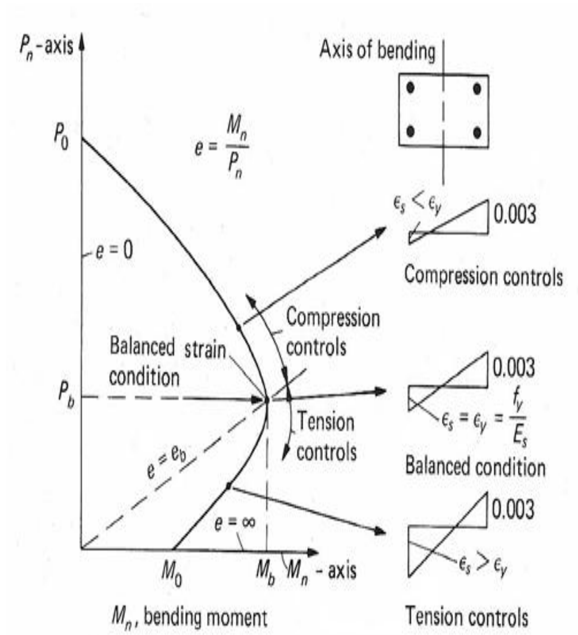

Figure 1: Typical Interaction Diagram

Interaction diagrams have two fold application, these diagrams could be used to assess the strength of a given column, on the other hand they could be effectively used to design a column to support a particular combination of external load and moment.

To develop interaction diagram, neutral axis will be assumed to be located at various positions and load and moment capacity corresponding to particular position of neutral axis were evaluated then the algebraic sum of load carrying capacity of steel and that of concrete gives the total axial load carrying capacity of the column, on similar lines total moment carrying capacity will also be calculated.

IV. Methodology

To proceed with first various locations of neutral axis were assumed and the capacity of column to carry the load and moment were evaluated for a particular location of neutral axis, as this procedure is repeated for various locations of neutral axis it is iterative, the procedure involves following steps.

A. *For the Case of Neutral Axis Situated Within the Section*

- P_{uc} = Capacity of concrete to carry axial load = 0.361 X f_{ck} X B X Xumax
  Where,
  f_{ck} = Characteristic compressive strength of concrete in N/mm^2
  B = Width **of** the section in mm Xumax = Depth of the NA from highly compressed edge in mm
- Strain in steel/concrete
  $$(\varepsilon_c) = 0.0035 \text{ X } \frac{\text{Xumax} - d_i'}{\text{Xumax}}$$
  Where,
  d_i' = distance of ith row of reinforcement from highly compressed edge
- Stress in steel (f_{sc})

Stress in steel can be evaluated by using Table 1, interpolation may be done for intermediate values.

Table 1: Values of Design Stress-Strain Curve for Hysd Bars

Stress Level	*f_y = 415 N/mm^2*		*f_y = 500 N/mm^2*	
	Strain	*Stress N/mm^2*	*Strain*	*Stress N/mm^2*
0.80f_{yd}	0.00144	288.7	0.00174	347.8
0.85f_{yd}	0.00163	306.7	0.00195	369.6
0.90f_{yd}	0.00192	324.8	0.00226	391.3
0.95f_{yd}	0.00241	342.8	0.00277	413.0
0.975f_{yd}	0.00276	351.8	0.00312	423.9
1.0f_{yd}	0.00380	360.9	0.00417	434.8

- Stress in concrete (f_{cc})
  When, $\varepsilon_c \geq 0.002$,
  $$f_{sc} = 0.446 \text{ X } f_{ck}$$
  When, $\varepsilon_c < 0.002$,
  $$f_{sc} = 446 \text{ X } f_{ck} \text{ X } \varepsilon_c \text{X } [1 - (250 \text{ X } \varepsilon_c)]$$
- Total load carrying capacity of column(Pu) = 0.361 X f_{ck} X B X D + Σ [Ast X (Stress in steel - Stress in concrete)]
- Capacity of concrete to carry moment
  (Muc) = 0.361 X f_{ck} X B X Xumax X [C.G – (0.416 X Xumax)]
  Where,
  C.G = Centre of gravity from highly compressed edge
- Total moment carrying capacity of column
  (Mu)= 0.361 X f_{ck} X B X Xumax X [C.G – (0.416 X Xumax)] + Σ [Ast X (Stress in
  steel - Stress in concrete) X lever arm distance]
  Lever arm distance = (C.G - d_i')

B. *For the Case of Neutral Axis Situated Outside the Section*

- Capacity of concrete to carry axial load (P_{UC})
  $$P_{UC} = C'1 \text{ X fck X B X Xumax}$$
  Where,
  fck = Characteristic compressive strength of concrete in N/mm2
  B = Width of the section in mm
  Xumax = Depth of the NA from highly compressed edge in mm
  C'1 = Stress block parameter corresponding to area of stress block
- Strain in steel/concrete (εc)
  $$\varepsilon c = \varepsilon_{cu} \text{ X } \frac{\text{Xumax} - d_i'}{\text{Xumax}}$$
  Where,
  εcu = Ultimate strain = 0.0035 – 0.75 X εmin
  εmin is evaluated as
  $$\frac{0.0035 - 0.75 \text{ X } \varepsilon_{min}}{(D+(K-1)D)} = \frac{\varepsilon_{min}}{(K-1)D}$$
- Stress in steel (fsc) using table 3.1
- Stress in concrete (fcc)
  When, εc> 0.002,
  $$fsc = 0.446 \text{ X fck}$$
  When, εc< 0.002,
  $$fsc = 446 \text{ X fck X } \varepsilon c \text{ X } [1 - (250 \text{ X } \varepsilon c)]$$
- Total load carrying capacity of column (Pu)
  $P_U = C_1'$ X fck X B X D + Σ [Ast X (Stress in steel - Stress in concrete)]
- Capacity of concrete to carry moment (Muc)

M_{UC} = C_1'X fck X B X Xumax X [C.G – (C_2' X Xumax)]

Where,
C.G = Centre of gravity from highly compressed edge
C_2' = Coefficient corresponding to centroidal distance of the stress block from highly compressed edge

- Total moment carrying capacity of column(M_U)
  M_U = C1' X fck X B X Xumax X [C.G – (C_2' X Xumax)]+Σ[Ast X(Stress in steel - Stress in concrete) X lever arm distance]

$$\text{Lever arm distance} = (C.G - d_i')$$

Table 2: Stress Block Parameters for NA Situated Outside the Section

$k = \frac{X_u}{D}$	*Coefficient* $C_1' = \frac{C_c}{f_{ck} \times B \times D}$	*Coefficient* $C_2' = \frac{Y_c}{f_{ck} \times B \times D}$
1.0	0.361	y
1.05	0.374	0.432
1.1	0.384	0.443
1.2	0.399	0.458
1.3	0.409	0.468
1.4	0.417	0.475
1.5	0.422	0.480
2.0	0.435	0.491
2.5	0.44	0.495
3.0	0.442	0.497
4.0	0.444	0.499

C. For the Case of Neutral Axis Situated at Infinity (eccentricity = 0)

$$P_u = 0.446 \text{ X } f_{ck} \text{ X } A_C + f_s \text{ X } A_{sc}$$

Where, A_c = Area of concrete

f_s= 0.79 X f_y for Fe 415 grade of steel

f_s = 0.75 X f_y for Fe 500 grade of steel

A_{sc} = Area of compressive steel

V. RESULTS AND DISCUSSION

In the present work interaction diagrams for cross (+) shaped columns were developed using the spread sheet, which could give interaction diagrams for cross(+) shaped, L – shaped, T-shaped, and rectangular section, the spread sheet developed will also have the provisions of modifying the grades of materials, spacing of reinforcement, size of the section et…, the interaction diagrams were developed based on the method suggested in SP:16, and IS : 456-2000, as described in the methodology in this paper, neutral axis positions were considered at various levels of the section and even outside the section for K > 1, as shown in table 2, and the

Values of P_U and M_U were obtained, the neutral axis positions were considered between the rows of reinforcement also to get a smooth, accurate and complete interaction diagram for the considered cross (+) shaped column.

Also the effect of varying grade of concrete and steel were studied by developing interaction diagrams for M20, M25, M30, M35, M40 grades of concrete with Fe415 and Fe500 grade steel, also a study has been carried out to establish the influence of varying spacing of reinforcement in comparison with constant spacing using interaction diagrams, the results were presented and discussed in this section.

A. Influence of Grade of Concrete with Fe 415 Grade Steel

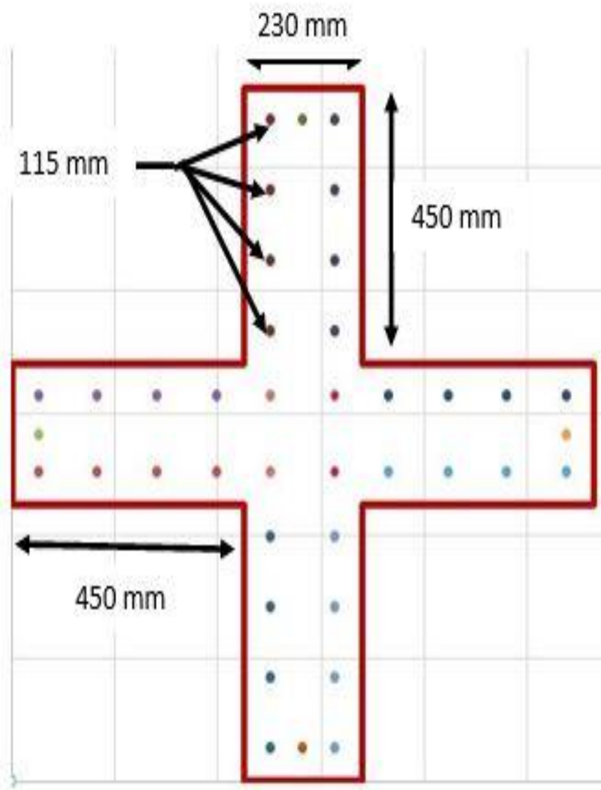

Figure 2: Cross (+) Shaped Column Considered to Study the Influence of Grade of Material

1) For M20 and Fe415 Grades

Problem description

- f_y= 415 N/mm^2
- f_{ck}= 20 N/mm^2
- Reinforcing bar diameter = 20mm
- Cover = 52.5mm

Tables for other grades were obtained in a similar fashion

2) For M25 and Fe415 Grades

Problem description

- f_y= 415 N/mm^2
- f_{ck}= 25 N/mm^2
- Reinforcing bar diameter = 20mm
- Cover = 52.5mm

Table 3: Load and Moment Capacity for M20 Grade Concrete and Fe 415 Grade Steel

Reinforcing bar diameter = 20mmCover = 52.5mmGRADE	*M20*	
DEPTH OF NEUTRAL AXIS 'X_U' (mm)	*LOAD CAPACITY (kN)*	*MOMENT CAPACITY(kN-m)*
∞ (PURE COMPRESSION)	8172.539746	0.0000
4520	8133.521962	25.6596
3390	8098.318541	44.1905
2825	8063.067919	62.0402
2260	7974.297505	100.9369
1695	7738.581313	200.4655
1582	7650.607614	239.0132
1469	7524.598715	288.6646
1356	7364.742839	354.5701
1243	7133.370285	444.3727
1186.5	6982.7615	503.7406
1130	6793.530303	580.9995
1077.5	6584.561469	673.6131
1020	6303.391787	776.8466
962.5	5991.295375	880.0819
905	5637.741585	984.2569
847.5	5236.526123	1089.4180
790	4747.586463	1195.3955
732.5	4218.132567	1294.8840
680	3699.737736	1374.1723
627.5	2800.922995	1332.9764
596.25	2232.035846	1298.0438
565	1651.87415	1250.9309
533.75	1042.444571	1194.7186
502.5	409.0297406	1127.1583
450	-751.223467	987.0489
397.5	-1506.065389	940.8042
340	-2240.556069	866.3968
282.5	-2694.779325	783.8703
225	-3043.522411	681.9035
167.5	-3363.992302	566.8249
110	-3693.526634	429.6172
52.5	-4107.880545	221.6753
0 (PURE TENSION)	-4535.202211	0.0000

3) For M30 and Fe415 Grades

Problem Description

- f_y= 415 N/mm^2
- f_{ck}= 30 N/mm^{2}4) For M35 and Fe415 grades

Problem description

- f_y= 415 N/mm^2
- f_{ck}= 35 N/mm^2
- Reinforcing bar diameter = 20mm
- Cover = 52.5mm

4) For M40 and Fe415 Grades

Problem description

- f_y= 415 N/mm^2
- f_{ck}= 40 N/mm^2
- Reinforcing bar diameter = 20mm
- Cover = 52.5mm

The interaction diagram has been plotted for these cases considering the various grades of concrete and the graph shows the comparison of load- moment resisting capacities of various grades of concrete for Fe 415 grade steel, the similar calculations were also carried out with Fe 500 grade steel and the interaction diagrams were plotted to study and compare the effect of varying grade of steel also, the results obtained were compared with that of ETABS-2015 and the conclusions were formulated.

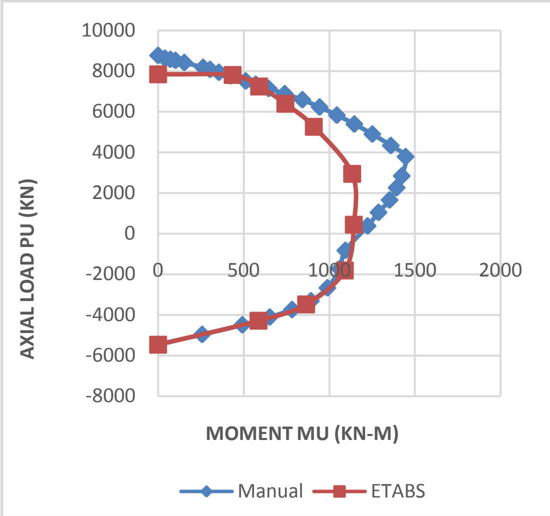

Figure 3: Comparison of Interaction Diagram for M20 Grade of Concrete Obtained from Spread Sheet with that of ETABS for Fe415 Grade Steel

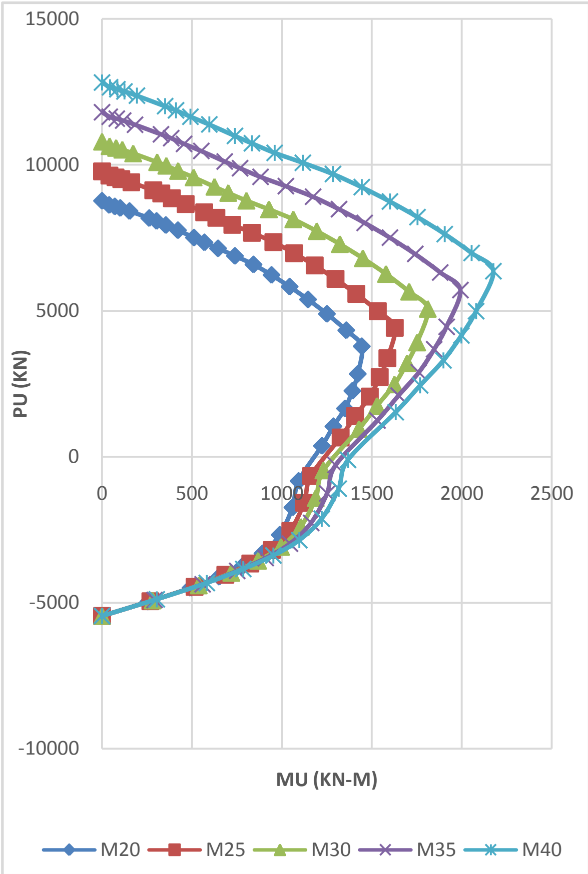

Figure 4: Interaction Diagram for M20, M25, M30, M35 and M40 Grade of Concrete with Fe415 Grade Steel

B. *Influence of Grade of Steel on Strength Characters of Cross (+) Shaped Columns*

Similar calculations were carried out to develop interaction diagrams with Fe500 grade steel and the results were compared with Fe415 grade steel.

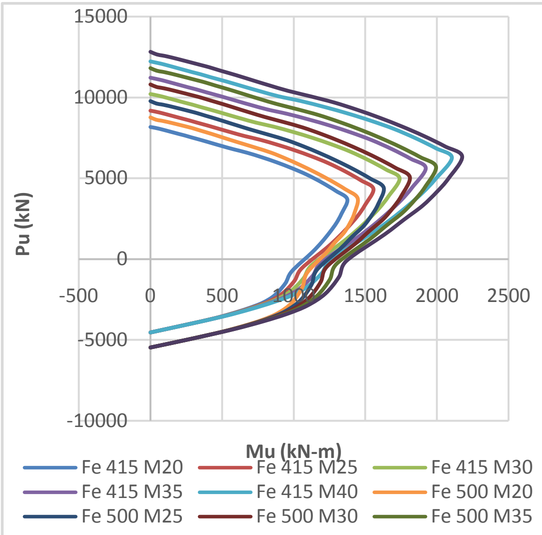

Figure 5: Comparison of Interaction Diagram for Fe415 and Fe500 Grade of Steel for M20, M25, M30, M35 and M40 Grades of Concrete

C. *Influence of Varying Reinforcement Spacing on Strength Characters of Cross (+) Shaped Columns*

1. *With Constant Spacing for Reference*

Problem description

- f_{ck} = 20 N/mm^2
- X left overhang = 550 mm Cover = 52.5 mm
- X right overhang = 550 mm
- f_y= 415 N/mm^2
- Y top overhang = 550 mm
- Y bottom overhang = 550 mm
- Constant spacing = 148.330 mm
- Cover = 52.5mm

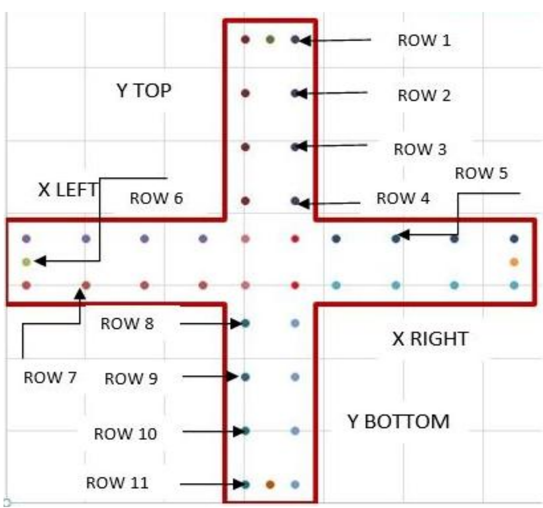

Figure 6: Cross (+) Shaped Column Considered to Study the Influence of Varying Spacing of Reinforcement

2. *Spacing between Row 1,2 and between 9,10 Reduced (Trial 1)*

Problem description

- Constant spacing = 148.330 mm
- Reduced spacing = 90mm
- Other specifications are similar to that of constant spacing case

3. *Spacing between row 1, 2, 3 and between 8, 9, 10 reduced(Trial 2)*

Problem description

- Constant spacing = 148.330 mm
- Reduced spacing = 90mm
- Other specifications are similar to that of constant spacing case

4. *Spacing between row 1, 2, 3, 4 and between 7, 8, 9, 10 reduced (Trial 3)*

Problem description

- Constant spacing = 148.330 mm
- Reduced spacing = 90mm
- Other specifications are similar to that of constant spacing case

5. *Spacing between row 3, 4 and between 7, 8 reduced(Trial 4)*

Problem description

- Constant spacing = 148.330 mm
- Reduced spacing = 90mm
- Other specifications are similar to that of constant spacing case

6. *Spacing between row 2, 3, 4and between 6, 7, 8 reduced(Trial 5)*

Problem description

- Constant spacing = 148.330 mm
- Reduced spacing = 90mm
- Other specifications are similar to that of constant spacing case

7. *Varying spacing between all rows (Trial 6)*

Problem description

- Spacing between row 1 and 2 = Spacing between row 10 and 11 = 148.330mm
- Spacing between row 2 and row 3 = spacing between row 9 and 10 = 130mmCover = 52.5 mm
- X right overhang = 550 mm
- f_y= 415 N/mm^2
- Y top overhang = 550 mm

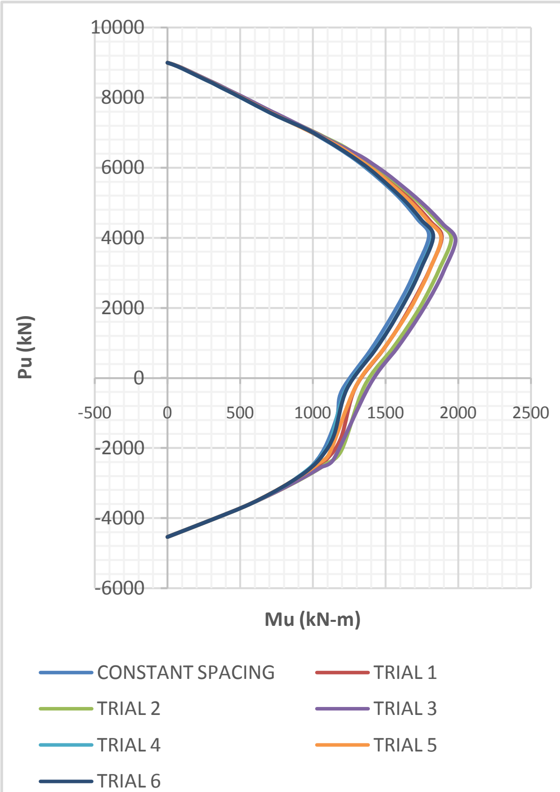

Figure 7: Interaction Diagram for Varying Spacing of Reinforcements

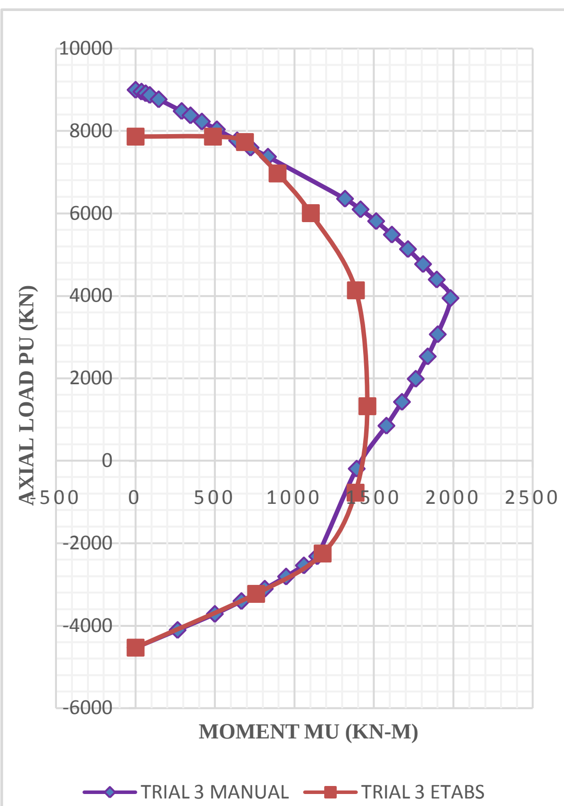

Figure 8: Comparison of Trial 3 of Varying Reinforcement Spacing Results with that of ETABS

For other variable spacing cases table is obtained in a similar fashion, and graphs were plotted as shown in Fig. 7

VI. CONCLUSION

1. With increase in grade of concrete or grade of steel the maximum load and moment carrying capacity of the cross (+) - shaped column increases.
2. Varying reinforcement spacing will not have any influence on maximum load carrying capacity, but the moment carrying capacity could be effectively increased by varying the spacing of reinforcement compared to constant spacing,
3. Decreasing order of moment carrying capacity with variable reinforcement spacing is trial 3, trial 2, trial 5, trial 6, trial 4, and constant spacing. This demonstrates that if the spacing between reinforcement layers at the outer periphery of legs away from central portion is reduced it results in an increase of moment carrying capacity of the section, and if the spacing between reinforcement layers near the central portion is reduced it results in decreased moment carrying capacity.
4. Comparison of results obtained from spread sheet with that obtained from ETABS has been done the results match well in the tension zone of the interaction diagram but the results of ETABS considerably deviates from manual calculations in compression zone because of consideration of Whitney's stress block parameter in ETABS.
5. A note given in Table H of SP:16 says that for construction of interaction diagram it is sufficient to consider value of K up to 1.2 only as it corresponds to minimum eccentricity, therefore ETABS considers the interaction up to 1.2D only but here the interaction diagram from spread sheet has been plotted for the value of k = 4, for information therefore there is a deviation from ETABS above minimum eccentricity.
6. Use of cross shaped columns with given percentage of steel can have a higher load and moment resisting capacity compared to that of rectangular sections.

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Table 4: Load and Moment Capacity for Constant Spacing of Reinforcing Bars

DEPTH OF NEUTRAL AXIS 'X_U' (mm)	*LOAD CAPACITY (kN)*	*MOMENT CAPACITY(kN-m)*
∞ (PURE COMPRESSION)	8993.179746	0.0000
5320	8950.593473	30.9046
3990	8911.816523	54.7270
3325	8873.006866	77.7570
2660	8774.614424	128.0756
1995	8517.662641	255.3952
1862	8421.011637	305.4095
1729	8281.006244	370.0847
1596	8104.794913	456.0535
1463	7850.12751	573.1465
1396.5	7682.196221	651.2376
1330	7470.362036	753.5871
1277.5	7281.598434	858.3629
1203.335	6971.708502	1006.3623
1129.17	6610.251481	1153.8180
1055.005	6199.427714	1301.2681
980.84	5708.477016	1447.7966
906.675	5135.230286	1596.6821
832.51	4528.277523	1720.5446
780	4055.861956	1797.9279
727.5	3139.670573	1711.6295
696.25	2566.326878	1649.0628
665	1985.871106	1573.5235
633.75	1385.452471	1487.4782
602.5	761.4632799	1390.6192
550	-355.3767168	1200.0037
497.5	-1095.166678	1166.1855
423.335	-2011.361959	1079.3445
349.17	-2567.587389	979.6099
275.005	-2959.734004	851.7499
200.84	-3307.203875	705.6974
126.675	-3662.414866	531.7352
52.51	-4107.880545	264.3973
0 (PURE TENSION)	-4535.202211	-0.0102