Cost Optimization of Elevated INTZE Water Tank

C.M. Indhudhar, Dr.K.P. Shivananda and Dr.J.K. Dattatreya

Abstract--- *A computer approach to the optimal design of elevated INTZE water tank is presented. The resulting to optimum design problems are constrained with non-linear programming problems. Parametric studies with resulting to different capacity of tank and grade of concrete have been carried out. The result of optimum design for INTZE water storage tank have been compared and conclusions drawn. For the minimum capacity and cost of the elevated circular water storage tank the following design variables are chosen as thickness of the cylindrical wall (X1), depth of the floor slab (X2) and depth of the conical floor beam (X3) and thickness of the bottom spherical dome(X4).The Constraints for the optimization are considered according to Standard Specifications. The optimization problem is characterized by having a combination of continuous, discrete and integer sets of design variables. The computer program is written in MATLAB.*

Keywords--- *INTZE Water Tanks, Optimization, Non-Linear Programming Technique, Tank Capacity.*

I. Introduction

WATER tanks are used to store water. Cost, shape, size and building materials used for constructing water tanks are influenced by the capacity of water tank. Shape of the water tank is an important design parameter because nature and intensity of stresses are based on the shape of the water tank. In general, for a given capacity, circular shape is preferred because stresses are uniform and lower compared to other shapes. Lesser stresses imply, lower quantities of material required for construction which brings down the construction cost of water tanks. The classification of water tank is as shown below.

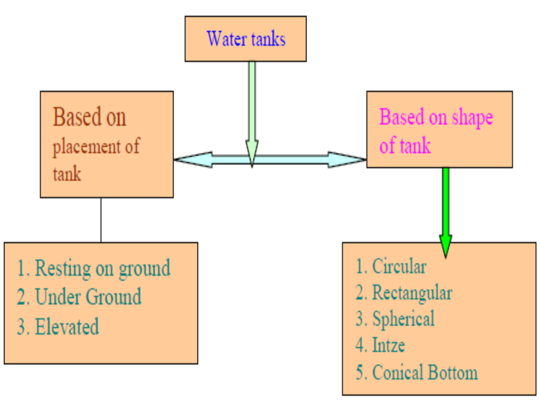

Figure 1: Water Tank Classification

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A cylindrical tank having spherical dome at the top and a smaller diameter dome at the bottom with cylindrical tank through a conical dome is called INTZE tank. In the case of large diameter elevated circular tanks, thicker floor slabs are required resulting in uneconomical designs. In such cases, INTZE type tank with conical and bottom spherical domes provides an economical solutions. The proportions of the conical and the spherical bottom domes are selected so that the outward thrust from the bottom dome balances the inward thrust due to the conical domed part of the tank floor.

II. Structural Elements of INTZE Tank

The various structural elements of an INTZE type tank comprises of the following:

1. Top spherical dome
2. Top ring beam
3. Circular side walls
4. Bottom ring beam
5. Conical dome
6. Bottom spherical dome
7. Bottom circular girder
8. Tower with columns and braces
9. Foundations

1. Top Spherical Dome

The dome at top is usually 100 mm to150 mm thick with reinforcement along the meridians and latitudes. The rise is usually l/5th of the span.

2. Top Ring Beam

The ring beam is necessary to resist the horizontal component of the thrust of the dome. The ring beam will be designed for the hoop tension induced.

3. Cylindrical Side Walls

This has to be designed for hoop tension caused due to horizontal water pressure.

4. Bottom Ring Beam

This ring beam is provided to resist the horizontal component of the reaction of the conical wall on the cylindrical wall. The ring beam will be designed for the induced hoop tension.

5. Conical Dome

This will be designed for hoop tension due to water pressure. The slab will also be designed as a slab spanning between the ring beams at top and the ring girder at bottom.

6. Bottom Spherical Dome

The floor may be circular or domed. This slab is supported on the ring girder.

7. Bottom Circular Girder

This will be designed to support the tank and its contents. The girder will be supported on columns and should be designed for resulting bending moment and Torsion.

8. Columns

These are to be designed for the total load transferred to them. The columns will be braced at intervals and have to be designed for wind pressure or seismic loads whichever govern.

9. Foundations

A combined footing is usually provided for all supporting columns. When this is done it is usual to make the foundation consisting of a ring girder and a circular slab

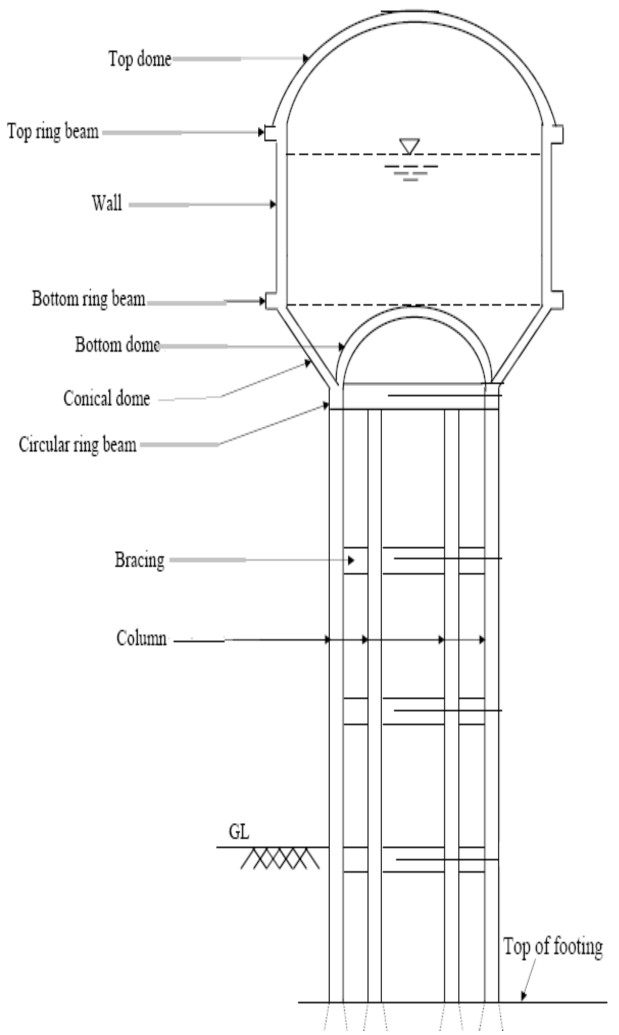

Figure 2: Elevation of INTZE Tank

III. Problem on INTZE Tank

Design an INTZE type water tank of 1000m3 supported on an elevated tower comprising of 8 columns. The base of the tank is 16 m above ground level. Adopt M-20 grade concrete and Fe-415 grade tor steel. The design of the tank should confirm to the stresses specified in IS:3370-1965 and IS:456-2000.

1. Data

Capacity of tank = 1000m^3

Height of supporting tower = 16m

Number of columns = 8

Depth of foundations = 1 m below ground level.

2. Permissible Stresses

M-20 grade concrete and Fe-415 grade tor steel, for calculations relating to resistance to cracking (IS: 3370)

σ_{ct} = 1.2 N/mm^2 σ_{cb} = 1.7 N/mm^2 σ_{st} = 150 N/mm^2

For strength calculations the stresses in concrete and steel are same as that recommended in IS: 456.

σ_{cc} = 5 N/mm^2 m = 13

σ_{cb} = 7 N/mm^2 Q = 0.897

J = 0.906

3. Dimensions of Tank

D = Inside diameter of the tank. Assuming the average depth = 0.75 D,

We have

$$\left(\frac{\pi D^2}{4} * 0.75D\right) = 1000m^3$$

D = 12m

Height of cylindrical portion of tank = 8 m

Depth of conical of Dome = 2m

Diameter of supporting tower = 8m

Spacing of bracing = 4m

4. Design of Top Dome

Thickness of dome slab = t = 100mm

Live load on dome = 1.5 kN/m^2

Self load of dome = (0.1×24) = 2.4 kN/m^2

Live load = 1.5

Finishes = 0.1

Total load = w = 4.0 kN/m^2

R = radius of dome

D = diameter at base = 12m

r = l rise = (1/6×12) = 2m

$$R = \left[\frac{(D/2)^2 + r^2}{2r}\right] = \left[\frac{6^2 + 2^2}{2 \times 25}\right] = 10m$$

Cosθ = (8/10) = 0.8

= 36°50'

Meridional thrust=$T_1 = \left(\frac{wR}{1+cos\theta}\right) = \frac{4*10}{1+0.8} = 22.22\ kN/m$

Circumferential force

$= wR\left(cos\theta - \frac{1}{1+cos\theta}\right) = 4*10\left(0.8 - \frac{1}{1+0.8}\right) = 10\ kN/m$

Mechanical Force $=\left(\frac{22.22*10^2}{1000*100}\right) = 0.22\ N/mm^2\ < 5N/mm^2$

$$\text{Hoop stress} = \left(\frac{10*10^2}{1000*100}\right) = 0.10\ N/mm^2$$

The stresses are within safe limits.

Providing nominal reinforcements of 0.3%

$$A_{st} = \left(\frac{0.3*100*1000}{100}\right) = 300\ mm^2$$

Provide 8 m ϕ at 160 mm centre to centre both circumferentially and meridional.

5. *Design of top Ring Beam*

Hoop tension = F1

$$= \left(\frac{T_1 * cos\theta * D}{2}\right) = \left(\frac{22.22 * 0.8 * 12}{2}\right) = 106.6\ kN$$

$$A_{st} = \left(\frac{106.6*10^3}{150}\right) = 710\ mm^2$$

Provide 8 bars of 12mm ϕ (A_{st}= 904 mm^2)

If Ac = cross sectional area of ring beam

$$\left(\frac{106.6 * 10^3}{A_c + 13 * 904}\right) = 12\ \therefore = 77082\ mm^2$$

Provide 300mm by 300mm size top ring beam, with 8 bar of 12mm ϕ as main reinforcement and 6mm ϕ stirrups at 200 centers.

6. *Design of Cylindrical Tank Wall*

Maximum hoop tension at base wall

$$F_t = \left(\frac{wh \cdot D}{2}\right)$$

Where w = density of water and

h = depth of water

$$\therefore F_t = \left(\frac{10 \times 8 \times 12}{2}\right) = 480\ kN/m$$

$$A_M = \left(\frac{480\times10^3}{150}\right) = 3200mm^2/\text{m height}$$

Provide 20mm ϕ at 180 mm centers on each face (A_{st}= 3492 mm^2) steel area required at 2 m below top is A_{st} = (2/8×3200) = 800mm^2.

Provide 10mm ϕ at 180mm centers on each face

If t= thickness of side wall at bottom

$$\left[\frac{480 \times 10^3}{1000t + (13 \times 3492)}\right] = 1.2\ \therefore t = 358mm$$

Adopt 400mm thick walls at bottom gradually reducing to 200mm at top.

Distribution Steel

At bottom, $A_{st} = \left(\frac{0.2\times400\times1000}{100}\right) = 800mm^2$

Provide 10mm ϕ at 10mm centres on both faces

Attop $A_{st} = \left(\frac{0.3\times200\times1000}{100}\right) = 600mm^2$

Provide 10mmϕat 250mm centres on both faces

7. *Design of Bottom Ring Beam*

Load on ring beam

(a) Load due to top dome = (Meridional thrust×sinθ)

= (22.22*sin 36.50) = 13.3kN/m

(b) Load due to top ring beam = (0.3×0.3×24) = 2.16kN/m

(c) Load due to cylindrical wall

$= \left(\frac{0.4+0.2}{2}\right) * 8 * 24 = 57.6\ kN/m$

(d) Self weight of ring beam (assuming a section 1.0m bt 0.6m)

= 1.2×0.6×24 = 17.28kN/m

Total vertical load =V1 = 91kN/m

Horizontal force =

H=v1cotθ=(91×cot 45°) = 91kN

Hoop tension due to vertical loads given by

$$Hg = \left(\frac{H*D}{2}\right) = \left(\frac{91*12}{2}\right) = 546\ kN$$

Hoop tensi0n sue to water pressure

$$Hw = \left(\frac{w*h*d*D}{2}\right) = \left(\frac{10*8*0.6*12}{2}\right) = 288\ kN$$

Total hoop tension = (Hg+Hw) = 546+288 = 834kN

$$Ast = \left(\frac{834*10^3}{150}\right) = 5560\ mm^2$$

Provide 18bars of 20mm (Ast = 5562mm^2)

Max. tensile stress

$$\left(\frac{834 * 10^3}{1200 * 600 + 13 * 5652}\right) = 1.05\ N/mm^2 < 1.2N/mm^2$$

Provide a ring beam of 1200mm wide by 300mm deep with 18 bars of 20mm ϕ and distribution bars of 10mm ϕ from cylindrical wall taken round the main bars as stirrups at 180mm centers.

8. *Design of Conical Dome*

Average diameter of conical dome $= \frac{12+8}{2}$ = 10m

Average depth of water = (8+2/2) = 9m

Weight of water above conical dome =π×10×9×2×10

= 5655kN

Assuming 600mm thick slab

Self weight= π×10×2.86×0.6×24 = 1280kN

Load from top dome, top ring beam, cylindrical wall and bottom ring beam = π×12×91 = 3430kN

Total load base of conical slab = 5655+1280+3430

= 10365kN

Load/ unit length = $V_2 = \frac{10365}{\pi\times8}$ = 412kN/m

Meridional thrust = T = V_2cosecθ = 413×cosec45° = 584kN

Meridional stress = $\frac{584\times10^3}{600\times1000}$ = 0.973N/mm^2< 5N/mm^2 (Safe)

Hoop tension in conical dome will be max at the top of the conical dome slab since diameter D is max at this section

Hoop tension =H = (p.cosecθ + q.cotθ)D/2

Water pressure = p = 10×8 = 80kN/m^2

Weight of conical dome slab per m^{2is} computed as,

q = 0.6×24 = 14.4kM/m^2

θ= 45°, D = 12m

H = (80×cosex45°+14.4×cot45°)*12/2 = 765kN

A_{st} = $\left(\frac{765\times10^3}{150}\right) = 5100$ mm^2

Provide 25mmϕ at 180mm centers (A_{st} = 5470mm^2) on both faces of slab

$$\text{Distribution steel} = \left(\frac{0.2\times600\times1000}{100}\right) = 1200\text{mm}^2$$

Provide 10mmϕat 130nn centers on both sides along the meridians

$$\text{Max. Tensile stress} = \frac{765\times10^3}{(600\times100)+(13\times5470)}$$

$$= 1.13\text{N/mm}^2<14.2\text{N/mm}^2$$

9. Design of Bottom Spherical Dome

Thickness of dome slab assumed as 300mm

Diameter at base = D = 8m

Central rise =r = (1/5×8) = 1.6m

If r= radius of the dome

(2R-r)*r = (D/2)^2

(2R-1.6)*1.6 = 4^2

R = 5.8m

Self weight of dome slab .

= (2×π×5.8×1.6×0.3×24 = 420kN

Volume of water above the dome

$$=\pi\times4^2/8+2*\left[\frac{2\pi\times5.8^2\times1.6}{3}-\frac{\pi\times4^2}{3}(5.8-1.6)\right] = 440\text{m}^3$$

Weight of water = 440×10 = 4400kN

Total load on dome = 420+4400 = 5320kN

Load /unit area = w=$\left(\frac{5320}{\pi*4^2}\right) = 106\ kN/m^2$

Meridional thrust = T_1 =$\left(\frac{wR}{1+cos\theta}\right)$

Cosθ = (4.2/5.8)= 0.724 ∴θ = 44.5°

$$T_1 = \frac{106*5.8}{1+0.724} = 357\ kN/mm^2$$

Meridional stress=

$$\left(\frac{357*10^3}{300*1000}\right) = 1.19\ N/mm^2$$

Circumferential force =

$$wR\left(cos\theta-\frac{1}{1+cos\theta}\right) = 106*5.6\left(0.724-\frac{1}{1+0.724}\right)$$

$$= 88.5\ kN/m$$

Hoop stress =$\left(\frac{88.5*10^3}{300*1000}\right) = 0.3\ kN/mm^2$ (safe)

Provide nominal reinforcement of 0.3%

$$A_{st} = \left(\frac{0.3\times200\times1000}{100}\right) = 600mm^2$$

Provide 12mm at 120mm centres circumferentially and along the meridians.

IV. Optimization of INTZE Tank

Optimization is the selection of a best element (with regard to some criteria) from some set of available alternatives. In the simplest case, an optimization problem consists of maximizing or minimizing a real function by systematically choosing input values from within an allowed set and computing the value of the function.

As the construction materials are getting extinct day by day it is important for the structural engineers to concentrate on optimum designing of the structures. With a special reference to structural problem it is always one of the minimizing or maximizing a certain specific characteristic of structural system like cost, weight, performance capability of the system depends on the problem.

This to be achieved without sacrificing any of the functional requirements like stresses deformation and load capabilities.

Thus, the optimization procedure must only be used to those problems where there is a definite need of achieving a quality product or competitive product.

The flowchart of the optimization program is as shown below

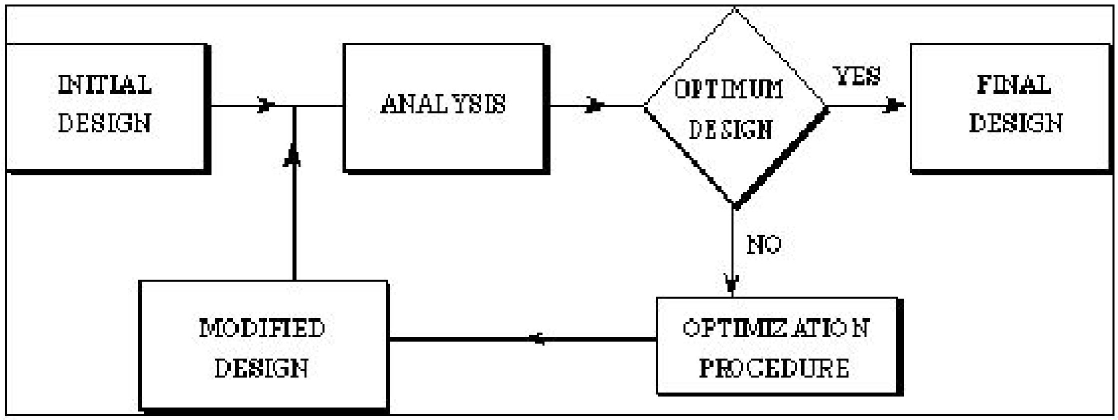

Figure 3: Flowchart of Optimization Program

An optimization or a mathematical programming problem can be stated as follows.

Find X = { X1, X2….Xn} …………..which minimizes f(X)

Subject to the constraints

$g_j(X) \leqslant 0$, j=1,2,…..m (Inequality constraint)

$h_j(X) = 0$ j=1,2,…p (equality constraint)

Where, X is an n-dimensional vector called the designed vector; f(x) is termed the objective function. The number of variables n and the number of constraints m and/or p need not be related in any way.

Methods of Optimization

a) Mathematical Programming Techniques
b) Stochastic Programming Techniques
c) Statistical Methods

a. Mathematical Programming Techniques

- Calculus Method
- Calculus Of Variations
- Nonlinear Programming
- Geometric Programming
- Quadratic Programming
- Linear Programming
- Dynamic Programming
- Integer Programming
- Stochastic Programming
- Separable Programming
- Multi Objective Programming
- Cpm & Pert
- Game Theory

b. Stochastic Programming Techniques

- Stastical Decision Theory
- Markov Processes
- Queing Theory
- Renewal Theory
- Simulation Methods
- Reliability Theory

c. Statistical Method

- Regression Analysis
- Cluster Analysis
- Design of Experiments
- Discriminate Analysis

The optimum cost design of elevated circular water tank formulated in is nonlinear programming problem (NLPP) in which the objective function as well as constraint equation is nonlinear function of design variables. The various methods available for the solution of NLPP are compared in brief and the advantages and limitation of the chosen method is discussed. The various subroutines used in the program are also discussed.

Methods for the Solution of the NLPP

1. Method of Feasible Directions
2. Sequential Unconstrained Minimization Technique
3. Sequential Linear Programming (SLP)
4. Dynamic Programming

Problem Statement

The first step in any optimization formulation is to identify the decision variables, and to define the objective function and the constraints that control the solution. The mathematical form of a single objective optimization can be stated as follows:

Find X^T = {X1, X2, ……, Xn} (1)

Such that Z = F(X) = Overall Cost →Minimum (2)

Subject to $g_j(X) \leq 0$ j = 1,m (3)

$X_i^{min} \leq X_i \leq X_i^{max}$ i= 1,n.

where, m is the number of inequality constraints (g), and n is the number of the design variables.

Depending on the nature of the objective function and the constraints, the general optimization problem posed by Eqs. (1) to (4) can be accordingly classified as linear or nonlinear mathematical models.

Design Variables

In the present study 4 variables are considered in the design; namely, thickness of cylindrical wall (X_1), thickness of bottom ring beam (X_2), thickness of the conical bottom dome (X_3), thickness of spherical bottom dome (X_4).

$$X^T = \{X_1, X_2, X_3, X_4\} = \{t_w, t_b, t_c, t_{bs}\}$$

Objective Function

The objective function is the total cost of concrete, and reinforcing steel required for the water tank, and can be described as follows:

Minimum F (X) = (cost of concrete * volume of concrete) + (cost of steel * volume of steel)

The concrete is typically measured by volume $V_c(X)$ whereas steel by weight $A_f(X)$.

It should be noted that $V_c(X)$ and $A_f(X)$ are both nonlinear explicit functions of design variables $\{t_w, t_b, t_c, t_{bs}\}$. Therefore, minimum and maximum bounds on the design variables are used during the search to avoid some combinations of these design variables that give rise to geometric infeasibility.

Table 1: Geometrical parameter of the INTZE water tank

References	K_1	K_2	K_3	K_4	K_5
Billig	0.125	0.6	0.625	0.125	0.188
Reynolds	0.125	0.66	0.625	0.125	0.188
Jain	0.125	0.30	0.70	0.14	0.179
Rao	0.125	0.25	0.60	0.09	0.238

Where

K_1= Rise to diameter ratio of top dome,

K_2= Height to diameter ratio of cylindrical portion,

K_3= ratio of diameters at the two ends of the conical dome,

K_4 = Rise to diameter ratio of bottom dome,

K_5= angle of the conical ome with horizontal.

Constraints

The constraints considered in the problem are:

1. The thickness of the cylindrical wall (t_w) and conical dome (t_c) is restricted to the limiting direct tensile stress as per IS: 456-2000.
2. The thickness of beam (t_b) and bottom spherical dome (t_{bs}) is restricted to permissible value of concrete.

Table 2: Permissible Concrete Stresses in Calculations Relating to Resistance to Cracking

Grade of concrete	Permissible stress in N/mm² tension		shear
	Direct	Bending	
M15	1.1	1.5	1.5
M20	1.2	1.7	1.7
M25	1.3	1.8	1.9
M30	1.5	2.0	2.2
M35	1.6	2.2	2.5
M40	1.7	2.4	2.7

V. Results and Discussions

The volume of the tank has been varied and the optimized thicknesses (variables) are obtained. The below table shows the optimized thickness for the given volume of tank. The minimum thicknesses of the cylindrical wall, bottom ring beam, conical bottom dome, spherical bottom dome were considered as 200mm, 200mm, 300mm and 200mm respectively

Table 3: Dimensions of INTZE Tank

Volume	t_w *(mm)*	t_b*(mm)*	t_c *(mm)*	t_{bs} *(mm)*
25	200	200	300	200
50	200	200	300	200
75	200	200	300	200
100	200	200	300	200
200	200	200	300	200
300	200	200	300	200
400	200	224	300	200
500	225	273	300	200
600	254	327	300	200
700	281	381	300	200
800	308	473	300	200
900	333	493	300	200
1000	357	550	300	200
1100	380	609	300	200
1200	403	668	300	200
1300	425	729	300	200
1400	447	790	300	200
1500	468	852	300	200
1600	488	915	300	200
1700	508	979	300	200
1800	528	1004	300	200

The values of the thickness of the conical bottom dome (t_c) and thickness of spherical bottom dome (t_{bs}) were minimum till the capacity was 6100m^3

VI. Conclusion

1. By using NLPP it is possible obtain optimized thickness for the given capacity of tank.
2. As the thicknesses of the elements of water tank are optimized, the dead load of the structure decreases, therefore an optimization on the self weight is also achieved.
3. By the optimized thickness, the cross sectional area and the amount of steel required decreases.

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